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The Art of Exposition

PRESIDENTIAL ADDRESS TO THE OPERATIONAL RESEARCH SOCIETY ON 27 NOVEMBER, 1959

THE RT. HON. THE EARL OF HALSBURY

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I ONCE attended a lecture by Sir Lawrence Bragg on the art of lecturing. In introducing his subject he commented: "Anyone who has the nerve to pick this subject for a lecture is asking for trouble." His fears I need hardly say were groundless, for what followed was a model of its kind.

I realize that in talking about the art of exposition I am running a similar risk. My excuse for the attempt is twofold. Firstly, the subject interests me; secondly, it has topical importance for the members of the Operational Research Society, one of whose preoccupations is the presentation of their work in intelligible form to colleagues or employers who may not share their technical and mathematical knowledge.

I know of few pleasures greater than that of watching the growth of understanding in a fellow-mind as the result of instruction and tuition for which one is responsible. In so far as pleasure can be altruistic, I would say that this must rank high among the list of those that are.

There is another pleasure, of course, in arranging one's own knowledge so that it forms a neatly ordered scheme based on minimum assumption and maximum compression so that it is as formally compact as possible. It is primarily egocentric. Other minds at one's own level of sophistication may, of course, enjoy the formalism but that is not the motive for which that formalization was undertaken. It is a selfish pleasure, but, as all work and no play makes Jack a dull boy, why should we forbid it? It may even be useful, albeit inadvertently.

There is a third pleasure to be obtained from writing about the history of a subject that is being taught. On the scale of altruism and egocentricity, it appears to stand intermediate between the extremes.

I will call these three kinds of exposition didactic, logical, and historical, and the first point I want to make is that each may involve a different order so far as the presentation of the subject's component elements is concerned.

Order is something unique. An ordered sequence cannot have two different orders. If, therefore, the didactic order of presentation does not naturally

coincide with the logical or historical order, something has to be sacrificed. That something has to be sacrificed is the recurrent theme of this paper.

The first questions that an expositor must therefore ask himself are "Why am I writing this?" and "Whom am I writing it for?"

These questions are extremely important when any kind of technical exposition is addressed to a non-technical reader, inasmuch as there is a much wider gulf in vocabulary to be bridged than in other situations.

If you will consider the study of classics, literature, history, political economy and, to a large extent, law, you will notice that they are taught very largely in the vocabulary of everyday life. There are, it is true, a few technical terms which occur in each by way of abbreviation or nicknaming, but they are usually avoidable and, if not, can be readily explained. If you have not studied poetry you may not know what a caesura is, but on being told that it is a natural pause occurring in the middle of a line, you will have no difficulty in assimilating the term. If you have not studied history you may not be familiar with the Donation of Constantine but it will be easy for you to follow a simple explanation of what it was—a forgery in terms of which the Papacy claimed to be heir-at-law to the Roman Empire in the West. Even the technical terms of law often carry a suggestion of what they may mean: "pleading", "indictment", "interrogatory", for example.

But the jargon of technology has no such ready connexion with everyday speech. Even an intelligent classicist who could guess correctly at the derivation of words such as "catalyst" or "entropy" would find his knowledge of little avail in using it to deduce the precise sense in which they are used technically, and on encountering nicknames such as Carathéodory's Principle or the Ergodic Hypothesis, no simple prose explanation could suffice to give him an inkling of what these names stood for.

In attempting any particular piece of exposition it is therefore essential to ask oneself these questions:

(1) Why am I writing this? For my own amusement or to convey information to some third party?

(2) To whom, if anyone, is it addressed? What sort of a person do I suppose him to be? What background of general science and mathematics may I assume? Am I a specialist reporting to a layman? Or a colleague writing for an equal? Or a junior reporting to a senior?

(3) How many words am I allowed for my purpose?

(4) What can I hope to say to the person I am writing it for in the number of words allowed? What do I want him to end up with as a result of reading what I am going to write?

(5) How am I going to do it?

Before I come to consider these points in detail, I want to dispose briefly of the reasons for which Arts graduates generally enjoy a better reputation than Science graduates as expositors of what they want to convey.

In the first place they usually have a much easier problem on their hands and can cut their teeth on easy stuff while they gain experience. The scientist only too often has to tackle something intrinsically difficult from the start.

In the second place, Arts graduates get more training in essay writing than scientists during the formative periods of their careers. They are required to write essays regularly and these essays are subject to criticism, not only for content but for style and manner of presentation, by academic staff who are themselves heirs to a stylistic tradition. Such essays as are written by science students will be judged more for their factual content than their style, and if stylistic failings are pointed out to them it will only rarely be by one who is himself a master of style.

Thirdly, there is practice. Writing is quite hard work. Writing well can be very hard work. To do hard and difficult work well, without prohibitive effort, needs a lot of practice, practice which for the most part scientists don't get for the good reason that they don't, in the literary sense, get called on to write very much.

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Ask any distinguished author—the type of man who has twenty or thirty works to his name—what he thinks of his early work. Many such authors would criticize their earlier work as showing faults of inexperience and immaturity. Consider, accordingly, how many scientists ever write twenty to thirty textbooks thereby acquiring the polish that distinguishes the mature author? Probably not one has ever done so in the history of scientific literature. Textbook authors probably produce one work in a career-time. Why then should one suppose that the author's first (and only) work would be, in the technical sense, well written?

With this much preamble let me consider the five questions I have asked above.

(1) *Why am I writing this? For my own amusement or to convey information to some third party?* This question is basic because there is no reason why a man should not write for his own amusement. But unless, when writing for his own amusement, he also writes *well*, there is equally no reason why anyone should bother himself to read it. And if the object of the exercise is to convey information to a third party and no information is conveyed, the exercise fails of its purpose. Much expository work is, in my opinion, bad because the author has never faced up to this issue. His finished effort is cast before his readers almost with the words: "Here's some pearls, you swine!" I have said earlier that didactic exposition is altruistic. Failure to be clear as to why one is expounding something is an essentially egotistic failure.

(2) *To whom, if anyone, is it addressed?* It may, of course, be no more than lecture notes intended to save students the trouble of taking their own. In these circumstances it may serve a useful purpose even if it is unintelligible to anyone not attending a course of the author's lectures. Duplicates of such notes are often sold for general consumption but in my opinion few of them are clear enough to merit general circulation. Aristotle's *Nichomachaean Ethics* has survived across twenty-two centuries by this process and though historians of

philosophy may be grateful for it, I doubt if professional philosophers get much satisfaction out of its obscurities. More commonly what issues from the pen of a lecturer, reader or professor, is a textbook, probably his only work and probably immature in consequence. It is often said of textbooks that they are not really intended to be read save as the accompaniments of a lecture course. Speaking as one who obtained his degree by exclusive reading of textbooks and by total non-attendance at lecture courses, I am not prepared to accept this excuse. Well-written textbooks can be clear as crystal; the reason that reading most of them is torture and that they are incomprehensible without first-aid from a lecturer or director of studies is that they are badly written. The ideal textbook is one that should be crystal clear to any student with the assumed background. So much for a master's writing, to be read, say, by his pupil. What of the reverse process in which a junior is reporting to his senior? To the question "What sort of a person do I suppose him to be?" there can be only one answer in modern times, to wit: "An extremely busy man". It is precisely in these circumstances that an egotistical junior, incapable of writing save for his own pleasure, merits, and probably receives, the undying hatred of his senior for the infliction of yet one more pile of *bumf* to take home at night. In writing for a senior it is well to assume that there was a time when the old man knew twice as much about the subject as oneself, since when he has forgotten half of it; but that it doesn't matter, as he will find his own way back to whatever he needs. He doesn't want a refresher course in the form of an essay. He wants a note of the current situation, of recommendations for action requiring his authority and the reasons for it, together with advance notice, below panic threshold, of anything liable to involve him in difficulties of his junior's making.

It is quite otherwise when tabling monthly reports for the perusal of colleagues. I was once responsible for a laboratory which turned out somewhat massive test results. To prevent my own thoughts falling into confusion and to ensure that my records were periodically straightened out, I wrote a monthly report. I wrote it primarily for myself. Since it had to be legible in future time, manuscript was contraindicated and I had it typed, thereby making copies available for colleagues on a research committee of which I was a member. Everyone took up a copy, but I don't think anyone ever read it; it wasn't intended for that purpose. I of course annexed a page of foolscap drawing attention to any results that mattered. Experimental results that matter sufficiently to require the notice of colleagues do not generally happen with a frequency that requires more than a monthly note.

So much for colleagues. You can treat them more or less as you please because they are in a position to retaliate. All's fair between equals. Blind them with science if you can and scatter triple integrals about your reports, but don't ever try that technique on a layman, especially if he is your senior.

Remember that that is exactly what he expects you to do and will react with exasperation at your first attempt: "What the hell is a Poisson distribution?

Why can't this so-and-so write English?" Queen Victoria never liked Mr. Gladstone because he addressed her as if she was a public meeting. There are ways of writing a report which evoke a like hostility in the reader. Face up to the realities of the situation. Is your putative reader a mathematician? The answer must be "yes" or "no". If he is not, do not try and squeeze in a little mathematics with a shoehorn just because *you* are interested in it. This is the egocentric sin: to write for yourself and not for your reader. Your difficulty may be that the subject appears inexplicable without some mathematics and much more interesting with them. Your task is to overcome that difficulty, not to run away from it. If you can't find a way round it, leave it alone and don't write the report. Go to your reader and discuss the difficulty with him. If he's a class of reader, pick a representative sample. If he's your senior, say: "I am due to give an account of my stewardship but it's a horribly technical one. Can you give me some help over what you really want to know?" He'll be delighted to do so, will begin to trust you at once, will happily leave all the technicalities to you, and will tell you what he wants to know which is probably quite simple. Can he, as *your* boss, tell *his* boss that you are earning your keep or are likely to do so within a reasonable period? If you are in operational research you are probably paying for yourself ten times over annually and all you have to do is to quote some results and put in for a rise! A week later he'll be telling *his* boss or his Board, "What I like about my boffins is that they just seem to work for the pleasure of it. If I could only get a few more like young so-and-so, we'd be increasing our dividend twice as often as we do".

Maybe your imagination boggles at the idea of anyone not being a mathematician. If imagination fails you, approach the problem intellectually and ask yourself "Am I an endocrinologist?" (Do you know what ICSH, ACTH and LH are or where the Islets of Langerhans are situated?) Continue, "If then I, being a mathematician, know nothing of endocrinology it is reasonable to suppose that there may be endocrinologists who know nothing of mathematics".

From the standpoint of technique much may be expounded with graphs that would cause alarm and despondency if expressed in analytical form. The National Institute of Industrial Psychology is one of my interests; I am Chairman of its Executive and Research Committee. My colleagues are concerned with policy from the standpoint of controlling the sort of research that is undertaken and ensuring that the results are of interest to industry. They include business men, engineers, trades unionists, administrators, representatives of D.S.I.R., and professors concerned with the academic aspects of the subject. Results have as often as not to be reported to them in terms of correlation coefficients and levels of significance, of all of which subjects the lay-members of the Committee are, broadly speaking, ignorant. We have therefore arranged a wall display in the committee room consisting of a number of scatter diagrams each representing some characteristic value of a correlation coefficient between familiar variables. We are accordingly in a position to report results simply by

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saying "The result looks like *that* (pointing at a scatter diagram) and the connexion between the variables is about the same as that between the price of milk and cheese on the same day". As a result of this simple approach harmony reigns between the technical and non-technical members of the Committee.

A refusal to recognize the limitations of readers accounts for much bad exposition of a semi-popular variety. Frequently some publishing house commissions a work with popular appeal to the educated. The author wants to use the Laplace Transform, say, because it is his pet toy. As a sop to Cerberus he therefore writes an introductory chapter in three parts:

(i) A refresher course in algebra

(ii) Elementary calculus

(iii) Introduction to the Laplace Transform.

He now feels free to mix his popular exposition with unpopular mathematics and the result is a thoroughly bad book. It will be bad because, in spite of the introductory chapter, it will be incomprehensible to the layman and superficial to the mathematician.

(3) *How many words am I allowed for my purpose?* This must be settled in advance. A textbook may be from 120,000 words upwards. A popular work will be about 60,000 words, the length of a light novel, say. A lecture will permit about 6,000 words and an article in a paper like *The New Scientist* from 1,500 to 2,000. Very occasionally one is asked to rewrite an essay review and given freedom up to 10,000 words, but these essays appear in high-class journals which don't make a profit, so that one is lucky to get £5 for the job! The length of a technical report or paper in the journal of a learned society depends upon the complexity of the subject-matter.

(4) *What can I hope to say to the person I am writing for in the number of words allowed?* This is a problem which many expositors fail to solve. One can usually say *something* in a sentence, e.g. "Biochemists are those who attempt to describe biological functioning in chemical terms". A second and a third sentence will not really enable one to say much more. To go significantly further one must have *substantially* more latitude. On going further, however, one meets a fundamental difficulty. Is the reader supposed to be a chemist or not? If not, then one must face the impossibility of introducing him to the whole of chemistry in part of a paper in order to render the whole intelligible with reference to only a part of chemistry.

There must be a limit *somewhere* at which biochemistry transcends the understanding of the non-chemically minded. Between that limit and another set by the possession of a degree in chemistry there is a sort of dead-zone in which nothing can be done for the layman. Outside these limits what one can say depends on the number of words allowed for saying it. One cannot hope to say in 1,500 words what would require 15,000. Something has to be sacrificed but the exact nature of the sacrifice has to be thought out.

The spoken word conveys less than the written word, for in a spoken lecture the audience cannot pause to rehear something they have not taken in as a reader can pause to reread an obscure paragraph. In communication engineering terms there must be more redundancy in a lecture than in a written essay. In a discursive didactically homogeneous lecture such as the present one, this difficulty is not so acute.

One point well made will be of more value than two points veiled in obscurity or doubt. If from this lecture tonight you take away the thought that expounding your results is as technically difficult as getting them and requires as much preparation and forethought, I shall have made the one point I seek to.

(5) *How am I to say it?* Here I can only offer a piece of negative advice. Do not suppose that your first thoughts on the matter will necessarily prove adequate. They may in special circumstances, but the odds are against it, so be prepared to have to rewrite what you have written again and again until it conforms to some ideal of perfection in your own mind. The reason that I cannot say more is that I have no fixed pattern to which I work. Sometimes I have brooded over a subject for years, waiting for an opportunity to go on record. When the opportunity comes, I may already have the subject worked out in my mind and can commit it to paper spontaneously in a form which needs little subsequent revision. On most occasions I have to write and rewrite to get it as I like it. *In extremis* I write a first draft in order to have something to bite on. It is usually a slapdash, bull-at-a-gate-affair, egocentric, diffuse, overlong and with some essential facts blank or filled in with guess-work. At this stage I have, so to speak, got the subject out of my system and can approach it dispassionately. Having done so I can start to try and identify the reader. This and considerations of length settle what I am going to omit and what I am going to emphasize, and I can go to draft 2 which represents the solid basis of what I am going to write. At this stage a lot of fact-checking is done as a result of which my own thinking begins to evolve and the presentation begins to rearrange itself in my mind. Consequently new paragraphs get inserted with pins in the course of the revision, old paragraphs get rewritten and scissors and paste get to work, cutting the whole thing up and rejoining it so that the presentation follows my latest ideas. The resulting mess could be called draft 3, and a terrible mess it often is at this stage. Draft 4 then emerges from draft 3 stylistically homogenized and rewritten in longhand for retyping. At this point I try and get finality over two matters of style and form originally specified by Aristotle:

(i) The thing must have a backbone. It must be about something in particular, not everything in general.
(ii) It must have a beginning, a middle and an end.

Draft 4 may then get "tried out on the dog", sometimes a member of my family enlisted for the purpose. They are all non-technical and if the verdict is "incomprehensible", then I must begin again because what one non-technical

person finds incomprehensible another will find the same. I do not wish you to suppose that everything I write needs this treatment. A broad spectrum of birth-pangs lies between the extremes of spontaneity and agony represented by the foregoing examples. Very often the document is not an essay but a committee report in which paragraphs from different hands are inserted. In this case I always try and ensure that the mass at stage 3 is rewritten in longhand and homogenized by a single author who can be relied on to write good English.

Notwithstanding all of which care and trouble, I have of course my failures like everyone else.

I want now to turn to two illustrations, one of bad and one of good exposition.

For bad exposition I will quote generally from my examination of some essays submitted in a competition intended to foster improvement in the presentation and exposition of technical matter by technical writers. The terms of reference included the following:

(1) *Intention.* The essays were to be readily understood by
  (a) Other scientists
  (b) Directors of industrial firms
  (c) Others interested in the advance of science and technology.

(2) *Requirement.*
  (a) About 3,000 words
  (b) The essay should disclose
    (i) The scientific background
    (ii) The experimental results
    (iii) The potential application of the project or process in industry.

(3) *Assessment.* The essay will be judged on
  (a) Technical content
  (b) Presentation
  (c) Style.

I found very little evidence that the great majority of the candidates had taken the slightest trouble to read their terms of reference, let alone to think about them. Many made no attempt to confine themselves to 3,000 words. A number merely submitted departmental research reports by permission of their employers, apparently unaware that the whole character of these reports was out of keeping with the objects for which the competition was instituted. There was little evidence that any of them had been revised so as to comply with requirements of presentation and style.

Now let me turn to the first part of the above terms. It presents a problem inasmuch as three *different* classes of readers are presupposed:

(a) *Other scientists.* This *must* mean scientists working in *other fields*; for an article written to be read by other scientists in the same field would merely be a technical paper.

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(b) *Directors of industry*. There is no reason to suppose that a director of industry has any scientific background at all. He may be a salesman, a lawyer or an accountant. Alternatively if he is in the scientific instrument industry he may know quite a lot about it.

(c) *Others interested in the advance of science and technology*. This does not state what the interest is supposed to be nor whether it is material or intellectual.

Now it has to be remembered that the essays were commissioned by judges in a competition, not by editors in a journal. Nothing prevented any candidate from making a note to the effect that he was exploiting the latitude given by the incompatible backgrounds of the supposititious readers so as to select *one* of them as his target. Only one candidate made any attempt to do so, however. Many others made a vague compromise between the requirements of (a), (b) and (c) and produced something both dull by criterion (a) and incomprehensible by criterion (b). The majority, however, seemed to have no particular reader in mind; they just wrote what came into their heads. The result was, to my way of thinking, very disappointing. It only indicated that most of the candidates hadn't a clue as to what was expected of them.

I now come to a more cheerful topic illustrating good exposition and for this purpose I have chosen what I regard as a modern masterpiece in textbook form: *The Structure of Physical Chemistry* by Sir Cyril Hinshelwood. Remembering that the author is not only President of the Royal Society but literate in seven languages, one would of course expect something good, but the excellence one discovers is not merely literary; it is expository excellence. The author makes you feel that he wants you to love the subject as he does, and ends up by making you do so. Anyone who really knew this book would know all he needed and he could learn it without attending a single lecture for it is all lucid from the first word to the last.

I want you to note the title: *The* Structure *of Physical Chemistry*. It gives you a foretaste of what you will get by reading it. You will in fact acquire inside your head the intellectual structure in terms of which physical chemists do their thinking. *The structure*: neither emasculated by formalism nor cluttered with obscuring detail. Let us turn to the introduction where the author tells us what he is going to do and why he is writing the book. In his own words:

> Physical chemistry is a difficult and diversified subject. The difficulty can, of course, be overcome by a suitable intensity of application and the diversity dealt with in some measure, by judicious specialization. This is very well as far as it goes, but leaves something to be desired, because there is, it is to be hoped, still room for a liberal occupation with wide studies, and this in a manner which goes beyond the polite interest of the dilettante. In the light of a good long spell of University teaching, however, I have the impression that this aspect of the matter is in some danger of neglect. I thought, therefore, I would like to write a book of moderate compass which, in no way competing with more formal works, should lay emphasis on the structure and continuity of the whole subject and try to show the relation of its various parts to one another. Certain themes or, one might almost say, leitmotifs, run through physical chemistry, and these would be used to unify the composition.

The treatment would be neither historical, nor formally deductive, but at each stage I would try to indicate the route by which an inquiring mind might most simply and naturally proceed in its attempt to understand that part of the nature of things included in physical chemistry. This approach I have ventured to designate humanistic. The proper study of mankind, no doubt, is man, but one of the greatest activities of man is to find things out.

Apart from the question of seeing the subject as a whole, there is that of seeing it with a sober judgement. It seems to me specially important in modern physical chemistry to be clear and honest about fundamentals. This is not so easy as it sounds. Some of the current working notions are expressed in words which easily become invested with a more literally descriptive character than they deserve, and many young chemists—this is my impression at least—are led to think they understand things which in fact they do not. Something simple and direct seems to be conveyed by words such as "resonance" and "activity", which is not legitimately conveyed at all. By certain descriptions, which it is easy to give, one is reminded of Alice: "Somehow it seems to fill my head with ideas—only I don't exactly know what they are." Many of the mathematical equations which serve important technical purposes in the modern forms of theoretical chemistry are of a highly abstract kind, but they have acquired a dangerous seductiveness in that they clothe themselves rather readily in metaphors. Occasionally it is salutary to regard this metaphorical apparel with the eyes of the child who surveyed the emperor's new clothes. I have done my best here and there to help the uninitiated reader keep in mind just what the content of theories amounts to.

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The author, you see, has faced up to the problem I put to you earlier. His mind is quite clear on three points:

(i) Why he is writing the book

(ii) The sort of book it is going to be

(iii) The class of readers whom it is intended to help.

The egocentric note is held in complete abeyance. Students are in danger of failing to see the wood for the trees. The book is to help them perceive their subject as a whole. It is not an egocentric expression of the author's personality as writer but an altruistic expression of sympathy with the personality of the reader.

The chapters that follow embody a most skilful blend of the logical and didactic orders. The subject not only unrolls in an order which makes it easy to learn, but each chapter follows logically on its predecessor as well. The order alone is a masterpiece of stage-management.

It is organized into parts, sub-divided into chapters. Each part commences with a short summary in beautifully clear prose of what that part is going to be about and of the relation that it bears to its predecessors. I would dearly like to quote from many of these summaries, but I must refrain from reasons of space and time and confine myself to two extracts which, however, I will quote *in extenso*.

Part I is entitled "The World as a Molecular Chaos", and Part II is entitled "Control of the Chaos by the Quantum Laws". When I first read the title of Part II, I remember my pleasure at the chosen phrase—"Control of the Chaos by the Quantum Laws". As a professional chemist I was, of course, familiar with all that Hinshelwood wrote of and my reading had included a more rather than a less than average amount of quantum theory, but I had never seen the connexion in just that way before. That single phrase seemed to illuminate

what I already knew, so that I saw it in a new light and the quality of my understanding was irreversibly altered. With this much preamble I now come to the two synopses.

Part I

Some of the ancient philosophers conceived the world to be made up of primordial particles in random motion, but their theories were not very fruitful since they lacked the necessary empirical basis. The formation of this was a long, complex, and far from obvious process.

Quantitative relations between the masses, and in certain cases the volumes, of substances which combine chemically establish the atomic theory as a scientific doctrine, and rather subtle coherence arguments reveal the distinction between atoms and molecules. Developments in physics lead to the recognition of heat as the invisible motion of the molecules themselves, and the kinetic picture of matter emerges.

Even in the first primitive version, this picture gives a satisfying representation of nature in many of its broad aspects. Molecules are envisaged as microscopic masses following the laws which Newtonian mechanics prescribe for macroscopic bodies. They are believed to be in chaotic motion and also to exert forces upon one another at small distances. Their motions tend to scatter them through all space: the forces to agglomerate them into condensed phases. The conflict between these two tendencies governs the existence of material systems in their various states, determines the range of stability of gases, liquids, and solids, and regulates the extent to which various possible combinations of atoms into molecules occur.

Reasonable estimates can be made of the absolute sizes, masses, speeds, and modes of motion of molecules, and coherent explanations can be given of many of the physical properties of matter in its different forms.

At this level of interpretation nothing is yet postulated about the forces, except that they manifest themselves in the measurable energy changes which accompany almost every kind of physical and chemical transformation. It is necessary, therefore, to describe the state of a chaotic system of particles primarily in terms of its energy. Such a description is provided in statistical theory.

In the world of molecular chaos various energy states are regarded as so many boxes occupied by molecules at random, and everything tends to that condition which may be realized in the largest number of ways. This idea leads to the definition of entropy, a function which measures the probability of a molecular assemblage, and to the laws of thermodynamics, which prescribe the conditions of equilibrium and the direction of possible changes in all such processes as expansion and contraction, melting and evaporation, and in actual chemical transformations. These laws themselves are closely related to empirical observations about heat-changes which can be used to provide an independent basis for them.

Given only an empirical knowledge of the energy changes accompanying atomic and molecular regroupings, the dynamical and statistical laws (or the equivalent thermodynamical principles) predict the relations of solids, liquids and gases for single substances and for mixtures, and also the dependence of chemical equilibrium upon concentration and temperature.

But the representation so made is limited in two ways. First, the forces remain unknown. The energy changes are measurable by experiment, but their nature remains unexplained. Secondly, the actual prescription of the condition of molecular chaos proves on closer inspection to present certain subtle problems which cannot be solved at this stage. If molecules are assigned to states according to the laws of probability, then what constitutes a state? In the first instance, equal ranges of momentum can be satisfactorily regarded as defining a series of states. But while this idea leads to valuable laws regarding changes in entropy, and the dependence of all kinds of equilibrium upon variables such as concentration and temperature, it leaves quite unsolved the question of the absolute position of chemical and physical equilibria.

Scientific explanations seek to describe the unknown in terms of the known. The first attempt at such an explanation of the material forms and relations of the world works in terms of particles which are themselves small-scale models of grosser objects. It goes a long way, but reaches a boundary beyond which it cannot pass.

Part II

As a result of a long series of intricate discoveries a solution is found to the problem of knowing where, in the absolute sense, this equilibrium lies between the different states of aggregation of matter and between the various configurations of atoms concerned in chemical transformations.

The first stage is the emergence of the quantum theory, according to which the energy of atomic or molecular systems varies discontinuously. The laws governing the various discrete series of possible energy states are discoverable from the study of such phenomena as specific heats and radiation. They evolve through different forms and are at length crystallized in rules whereby the energies are defined in terms of the permissible solutions of a semi-empirical differential equation, known as the wave equation.

The foundation of this equation is the discovery that on the scale of electronic or atomic phenomena particles obey dynamical laws which are neither precisely those followed by macroscopic masses nor yet those followed by light waves, but are of a special kind.

The new rules impart to the theory an abstract basis. Atoms and molecules can no longer be regarded as small-scale versions of ordinary objects. Furthermore, we have to conclude that there is no physical sense in treating different permutations of individual atoms or molecules within a given energy state as theoretically distinguishable systems.

Given these apparent sacrifices of the primitive simplicity, energy states become, in compensation, like so many exactly defined boxes, the allocation of molecules to which can be treated by the laws of probability. Absolute equilibria are now seen to be governed by the interplay of two major factors: on the one hand, the tendency of atoms and molecules to assume a condition of minimal potential energy, and on the other hand, their tendency to fill impartially all energetically equivalent states.

With the equilibrium of solid and vapour, for example, the potential energy factor favours condensation of all the molecules to solid. But in the vapour the range of possible energy levels is much greater and molecules populate states according, as it were, to the housing conditions, quantum levels representing in effect accommodation. Expressed in another way, the atoms and molecules escape from the restraints imposed by the forces acting upon them in so far as they achieve fuller self-realization in conditions where more modes of motion and more quantum levels offer them opportunity. The quantum theory having given exact formulations of the accommodation ranges, these statements can be translated into precise terms which lead to a quantitative treatment of all types of equilibrium.

At this stage the nature of the forces and of the interaction energies still remains unknown.

For those of you who are professional mathematicians rather than chemists or physicists I would recommend a study of Forsyth's *Theory of Function of a Complex Variable* as an object lesson in exposition.

It is longer than any work that a modern publisher would be prepared to support, the present costs of paper and typesetting being what they are. It remains a great work in my opinion because the author leads one on continuously throughout the course of an exciting adventure and never ceases to hold one's hand on the way. By contrast modern formalized and compressed treatments of the same subject are about as intellectually stimulating as a breakfast of wet blotting paper.

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What is the relevance of this to operational research? I think it is very relevant because operational research has to be explained and expounded to those who will benefit from it in order that they may employ it and do so. If operational research workers do not expound their work to actual or potential employers, it is quite certain that no one else will do so. And if they expound it badly or obscurely so that their techniques are undervalued and their skills lie fallow, it is the nation that will be the loser.

The exposition of operational research is attended by certain intrinsic difficulties. Nothing issues from it in the form of hardware which can be pointed to as "the result". Results show up in statistics of performance, rarely unmixed with, and often inseparable from, results derived by other methods. The merit of the operational researcher's work may be assessable only by accountants, themselves unable to follow his methods. By contrast colleagues in the engineering and physical sciences who may be employed alongside him and who are qualified to follow his line of thought may have no access to information which would confirm the success of his activities.

Many technical workers suffer from some of these difficulties; they are enhanced for operational research workers inasmuch as their technique is essentially mathematical and mathematics is the most forbidding and least popular of all subjects outside the ranks of its practitioners. I know many soldiers, sailors, tinkers and tailors who are quite respectable amateur engineers. I cannot recall one amateur mathematician.

If in these circumstances operational research workers expound their results so as to fall into any of the pitfalls of which I have spoken, the result can only be unfortunate.

May I at this stage say a few words about the problem of reporting to a *Board*, not a managerial Board responsible for local operations but the sort of non-executive Board that has overall responsibility to shareholders. You may only be called on to address such a Board once in a lifetime, but on that one occasion it is incumbent on you to be a credit to your profession. You will probably have at most a quarter of an hour. Don't make the mistakes of either trying to teach your audience about operational research in that time, or of condescending to them because they don't know what it is already. One member may be a solicitor, another a chartered accountant, a third a merchant banker, a fourth a sales director from another industry, and so on. You must assume that they have only two features in common. The first is ignorance of operational research. The second is a highly developed shrewdness in judging of men and affairs. They will be quick to assess your capacity for seeing your own subject in perspective, for distinguishing the wood from the trees. In these circumstances be simple and non-technical, and don't get into an argument about technicalities. If you are on a sticky wicket the chairman or the technical director will probably come to your rescue if you give them a chance. If what they say isn't up to your own standard of rigour, let well alone and shut up! You may know more of

operational research than they do, but they know more about their colleagues than you do.

Now to some more specific problems.

I note that some difficulty has attended the definition of what operational research *is*. I was myself embroiled in similar difficulties over the definition of automation a few years ago. It was distressing to find that people will embark on definitions without any previous instruction in philosophy or formal logic, where definition is itself the subject of discussion. It is because expositors are floundering about in unfamiliar territory that they get into these difficulties. There is no excuse for them.

A definition may be "denotational" as in a dictionary. In this sense one may say that a dog is a quadruped that wags its tail and barks. If you know the meaning of "quadruped", "wag", "tail" and "bark", you then know what a dog is. Alternatively a definition may be "ostensive", as when one shows an object to a child and names it so that the child may come to associate the object with its name. In this sense one points at a dog and says "dog". Every vocabulary must contain *some* names ostensively defined, and the object of the exercise is merely to enable people to use words correctly—that is, in a similar sense to that in which other people use them. From this point of view it is not necessary to say more of the words "operational research" than that "their meaning will become clear to anyone who reads what follows". This is to employ the ostensive approach. The game of reducing a vocabulary to the minimum number of ostensive elements is one we can leave to lexicographers.

In expounding work to laymen we must, I think, use the method of the catalogue so that the reader has before him a long list of problems which, though they may appear physically diverse, are nevertheless structurally similar and therefore describable by a single mathematical formula. We want the layman to recognize a queue under its diverse manifestation so that by thinking of service times and waiting periods he can call us in to help him with problems that have occurred to him as problems, not as Acts of God, or the Queen's Enemies.

Distributions—Poissonian, binomial and normal—require to be explained by examples from occurrences and by graphs for the purpose of illustrating properties. There is no reason why the layman should not learn to recognize a distribution that is Poissonian. It is as unnecessary for him to know about its mathematical description as it is for a spin-bowler to know the mathematics of gravitation and aerodynamics. Above all, this illustration must be in concrete terms and *copious*. An abstract presentation coupled to a solitary instance will achieve nothing.

Results need to be presented graphically and the graphs must be described in clear prose stating what they represent. Not even the simplest terms such as maximum and minimum should be hurried over. They may present unexpected difficulties to some layman and deserve a paragraph at their first occurrence and

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a sentence on subsequent occurrences. And, of course, graphs must be clearly labelled in practical units so that they come alive for the reader.

Finally, results must be quoted wherever possible in monetary terms. We must blow our trumpet a bit louder whenever we save something. These are perhaps rather vulgar considerations to be put forward in a presidential address to a learned society, but they spring from one simple proposition that has inspired the present work. The object of exposition is the transfer of information and understanding from the expositor to his readers or audience. If, in fact, nothing is conveyed, then the exercise has failed in its purpose. The most common reason for failure is attempting too much because the problem has not been seen as such and its associated difficulties correspondingly underestimated. Egotism is usually at the heart thereof.

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Some Reliability Models of Production Lines, with Special Reference to Computer Operation and Scheduling*

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A situation is considered where a machine or a production line, for example an electronic computer or a data-processing system, has alternate "good" and "bad" periods of operation. Two types of model are described which can be used to tackle this kind of situation and these form an approach to the practical problem of scheduling work through such a system. The two aspects considered are the probability of a preassigned schedule of operation being completed within some critical time, and the optimum assignment of the length of the scheduled operations.

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A SITUATION is considered where a machine or a production line, for example an electronic computer or a data-processing system, has alternate "good" and "bad" periods of operation. Two types of model are described which can be used to tackle this kind of situation.

The first problem considered is the estimation of the probability of fulfilling, inside a total operating period of length T, a preassigned schedule of operations, taking time S to complete if no faults or stoppages occur. Theoretical methods are derived for the approximate evaluation of this probability, about which several theorems are stated *without proofs* (for their proofs, see Mayne[1]). An attempt to apply the first reliability model to evaluate this probability for an actual electronic computer is described briefly.

The second problem considered is the optimum assignment of the length of the scheduled operations. Here, a balance must be found between too much caution (putting on too short a sequence of operations and letting the system be idle for too long) and too tight a schedule (running too great a risk of not completing the operations on time). No simple analytical solution has been found to this sort of problem, but numerical evaluation of approximate optima is straightforward and should be easy to carry out on an electronic computer.

Finally, some more general problems are considered briefly.

1. THE FIRST RELIABILITY MODEL

1.1. *Statement of the problem and specification of the model.* Suppose that a machine or a production line has alternate "good" and "bad" periods of operation. Let a job have to be carried out, which would be completed in time S if there were no delays due to the presence of "bad" periods. In practice, the job will take a time t, at least equal to S, to finish, because such delays might

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arise. The manager may wish to make every effort to ensure that t does not exceed some critical time T; if t *does* exceed T, then the job is said to be *completed off-schedule*. In order to formulate a suitable production policy, the manager will wish to calculate the *off-schedule probability* $P(S, T)$, which is the probability that t exceeds T. Thus it is the probability that a job, which can be completed in time S when delays are absent, is not completed after the machine or production line has been operating for time T.

In the most general form of the first reliability model, let $p_r(S)$ be the probability that there are r stoppages in the course of carrying out a job of length S. Assume that work is started on a job at an instant which lies inside a "good" period. Thus, for given S, the operating period, during which this job is carried out, contains $(r+1)$ "good" periods, of total length S, interspersed with r "bad" periods. $(r = 0, 1, 2, \ldots .)$ Let $H_r(\,.\,)$ be the cumulative distribution function of the total length of r consecutive "bad" periods. Two special cases, believed to be of great practical importance, are now defined.

In *Special Case* 1, it is assumed that the "good" and "bad" periods all have mutually independent distributions, that all "good" periods have the same cumulative distribution function, and that all "bad" periods have the same cumulative distribution function. Let $G(u)$ be the probability that a "good" period has length $\leqslant u$. Let $H(u)$ be the probability that a "bad" period has length $\leqslant u$.

Special Case 2 is the example of Special Case 1 where $G(u) = 1-e^{-au}$ (i.e. $p_r(S) = e^{-aS}(aS)^r/r!$ $(r = 0, 1, 2, \ldots)$, by a standard result of probability theory), and where $H(u) = 1-e^{-bu}$.

1.2. *Calculation of the off-schedule probability for the general case.* Two methods for approximating to $P(S, T)$ are considered here. The first method is to find the Laplace transform $\Phi(S, p)$ of $F(S, X)$ (which is defined as $[1-P(S, S+X)]$) and then to invert it numerically. For some remarks on the numerical inversion of Laplace transforms, see Appendix I. $F(S, X)$ can be interpreted as the probability that the total length x of the stoppages, experienced while carrying out a job which takes time S to complete without stoppages, is at most equal to X.

By definition,
$$\Phi(S, p) = \int_0^\infty e^{-pX}\, d_X F(S, X).$$

Let
$$W_r(p) = \int_0^\infty e^{-pu}\, dH_r(u),$$

$$W(p) = \int_0^\infty e^{-pu}\, dH(u),$$

$$p(S, z) = \sum_{r=0}^{\infty} p_r(S)\, z^r.$$

Theorem 1. (a) In the most general specification of the first reliability model,

$$\Phi(S,p) = \sum_{r=0}^{\infty} p_r(S)\, W_r(p).$$

(b) In any example of the first reliability model where all the "bad" periods are independently and identically distributed, with cumulative distribution function $H(.)$,

$$\Phi(S,p) = p(S, W(p)).$$

The *second method of approximating to* $P(S,T)$ is to use the values of the cumulants of t (the time taken to complete the job), and to apply an approximation based on that of Cornish and Fisher.[3] An outline of some results about such approximations is given in Appendix II.

Let $K_i, K_i(S), K_i(x), K_i(t)$ be the respective cumulants of: the distribution with cumulative distribution function $H(.)$; the distribution $p_r(S)$ $(r = 0, 1, 2, \ldots)$; the distribution of x (which has cumulative distribution function $F(S,X)$); the distribution of t (which has cumulative distribution function $P(S,T)$); where S is a given constant. Assume that all these cumulants exist; this seems to be true in practical examples.

Theorem 2. In any example of the first reliability model where all the "bad" periods are independently and identically distributed, with cumulative distribution function $H(.)$,

(a)
$$K_1(x) = K_1(S)\,K_1,$$
$$K_2(x) = K_2(S)\,K_1^2 + K_1(S)\,K_2,$$
$$K_3(x) = K_3(S)\,K_1^3 + 3K_2(S)\,K_1\,K_2 + K_1(S)\,K_3,$$
$$K_4(x) = K_4(S)\,K_1^4 + 6K_3(S)\,K_1^2\,K_2 + K_2(S)\,(4K_1\,K_3 + 3K_2^2) + K_1(S)\,K_4,$$
$$K_5(x) = K_5(S)\,K_1^5 + 10K_4(S)\,K_1^3\,K_2 + 5K_3(S)\,K_1(2K_1\,K_3 + 3K_2^2) + 5K_2(S)\,(K_1\,K_4 + 2K_2\,K_3) + K_1(S)\,K_5,$$
$$K_6(x) = K_6(S)\,K_1^6 + 15K_5(S)\,K_1^4\,K_2 + 5K_4(S)\,K_1^2(4K_1\,K_3 + 9K_2^2) + 15K_3(S)\,(K_1^2\,K_4 + 4K_1\,K_2\,K_3 + K_2^3) + K_2(S)\,(6K_1\,K_5 + 15K_2\,K_4 + 10K_3^2) + K_1(S)\,K_6.$$

(b) $K_1(t) = S + K_1(x)$; for $i \geqslant 2$, $K_i(t) = K_i(x)$.

A generalization of Theorem 2, valid for the most general specification of the first reliability model, is given in Mayne[1].

1.3. *Calculation of the off-schedule probability for special case* 1

Let
$$M = \int_0^{\infty} u\, dG(u), \qquad X(q) = \int_0^{\infty} e^{-qu}\, dG(u).$$

Assume first that the start of the schedule period of length T is chosen at random from the set of instants lying in "good" periods. Then the following three theorems can be proved (see Mayne[1]).

Theorem 3. If Special Case 1 holds, then

$$\int_0^\infty e^{-qS}\,dS \int_0^\infty e^{-pX} F(S,X)\,dX = \frac{1}{pq} - \frac{[1-W(p)][1-X(q)]}{Mpq^2[1-W(p)X(q)]}.$$

Theorem 4. If Special Case 1 holds and if $H(u) = 1-e^{-bu}$, then

$$\int_0^\infty e^{-qS} F(S,X)\,dS = q^{-1} - M^{-1}q^{-2}[1-X(q)]\exp\{-bX[1-X(q)]\}.$$

Theorem 5. If Special Case 1 holds and if $G(u) = 1-e^{-au}$, then

$$\Phi(S,p) = \exp\{-aS[1-W(p)]\}.$$

Theorem 5 was originally derived by Takács[4], who has also derived some results relevant to Special Case 1 in general.

Now assume, instead, that the start of the schedule period of length T is chosen at the beginning of a "good" period. Then Theorem 5 still holds, but Theorems 3 and 4 must be replaced by Theorems 3* and 4* respectively.

Theorem 3*. If Special Case 1 holds, then

$$\int_0^\infty e^{-qS}\,dS \int_0^\infty e^{-pX} F(S,X)\,dX = \frac{[1-X(q)]}{pq[1-W(p)\,X(q)]}.$$

Theorem 4*. If Special Case 1 holds and if $H(u) = 1-e^{-bu}$, then

$$\int_0^\infty e^{-qS} F(S,X)\,dS = q^{-1}[1-X(q)\exp\{-bX[1-X(q)]\}].$$

Further results about Special Case 1 are given in Mayne[1].

1.4. *Calculation of the off-schedule probability for special case* 2

Let $$X = T-S, \quad c = (aS)^{\frac{1}{2}}, \quad d = (bX)^{\frac{1}{2}}, \quad k = d-c.$$

Let $$I_r(x) = \sum_{s=0}^{\infty} (\tfrac{1}{2}x)^{r+2s}\, s!\,(r+s)!^{-1},$$

which is a Bessel function, for which some tables of values are available.

Theorem 6. If Special Case 2 holds and if $T \geqslant S$, then

(a) $$\Phi(S,p) = e^{-aSp/(p+b)}.$$

(b) $$P(S,T) = e^{-(c^2+d^2)} \sum_{r=0}^{\infty} c^r d^{-r} I_r(2cd) \quad (c \leqslant d)$$

$$= 1 - e^{-(c^2+d^2)} \sum_{r=0}^{\infty} c^{-r} d^r I_r(2cd) \quad (c \geqslant d)$$

$$= 2ce^{-c^2} \int_d^\infty e^{-u^2} I_1(2cu)\,du.$$

(c) For $c = 0$, i.e. $k = 0$,

$$P(S,T) = \tfrac{1}{2}\{1 - e^{-2c^2} I_0(2c^2)\}.$$

(d) For $c = 0$, $P(S,T) = 0$.

(e) For $c = \infty$, $P(S,T) = 1 - \Phi(2^{\frac{1}{2}}k)$,

where $$\Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt,$$

which is the standard normal cumulative distribution function, as tabulated in standard statistical tables.

(f) For any given value of k, and for $0 \leqslant c \leqslant \infty$, $P(S,T)$ increases as c increases; in particular, $P(S,T) < 1 - \Phi(2^{\frac{1}{2}}k)$, if $c < \infty$.

(g) For any given value of c, for which $0 \leqslant c \leqslant \infty$, $P(S,T)$ decreases as k increases.

Various approximations to $P(S,T)$ are described in Mayne[1]. Here, mention is made only of those approximations which are based on the results of Appendix II, as they seem to be the approximations most often useful in practical applications.

The first of these approximations is

$$P(S,T) \doteq 1 - \Phi(2^{\frac{1}{2}}k^*),$$

where $$k^* = k + (4c)^{-1} - 3k^2(4c)^{-1}(6kc + 4k^2 + 1)^{-1}.$$

It is accurate to within 1 per cent if $c \geqslant k, k \geqslant 1$. It is also fairly good if $c \geqslant \frac{1}{2}k$, $k > 1$. It *underestimates* the true value of $P(S,T)$.

The second of these approximations is

$$P(S,T) \doteq 1 - \Phi(2^{\frac{1}{2}}k^{**}),$$

where $$k^{**} = k + \frac{24(k^2+1)c + 3(4k^2+3)k}{96(k^2+1)c^2 + 12(8k^2+7)kc + 2(8k^4+k^2-4)}.$$

It has about the same accuracy as the first approximation. It *overestimates* the true value of $P(S,T)$.

The third of these approximations is cruder, but is very simple to obtain. It is $P(S,T) \doteq 1 - \Phi(2^{\frac{1}{2}}k')$, where $k' = k + (4c + 2k)^{-1}$. It *overestimates* the true value of $P(S,T)$.

For some details of other approximations, see Mayne[1].

The following table gives the values of $P(S,T)$ for $c = 0{\cdot}1, 0{\cdot}2, 0{\cdot}5, 1, 2, 5, 10, \infty$; $k = 0(\frac{1}{2})5$, where $c = (aS)^{\frac{1}{2}}$ and $k = b^{\frac{1}{2}}(T-S)^{\frac{1}{2}} - (aS)^{\frac{1}{2}}$. These values are given to three significant figures where $P(S,T) < 0{\cdot}1$, and to four significant figures where $P(S,T) \geqslant 0{\cdot}1$; the last figure may have an error of one unit in a few of the entries. Almost all the entries have been calculated by at least two methods; the resulting approximations, when carried out inside their region of validity, usually agree, to the accuracy used here, but sometimes differ by one unit in the last figure.

TABLE 1. TABLE OF $P(S, T)$

k \ c	0·1	0·2	0·5	1	2	5	10	∞	c / k
0	0·00985+	0·0377	0·1775−	0·3457	0·4283	0·4717	0·4859	0·5000	0
½	0·00696	0·0243	0·0915	0·1551	0·1928	0·2193	0·2292	0·2398	½
1	0·00299	0·00956	0·0302	0·0472	0·0594	0·0696	0·0739	0·0787	1
1½	0·000779	0·00231	0·00634	0·00948	0·0121	0·0146	0·0157	0·0170	1½
2	1·24 $\times 10^{-4}$	3·41 $\times 10^{-4}$	8·40 $\times 10^{-4}$	0·00123	0·00158	0·00195	0·00212	0·00234	2
2½	1·19 $\times 10^{-5}$	3·08 $\times 10^{-5}$	6·98 $\times 10^{-5}$	1·00 $\times 10^{-4}$	1·30 $\times 10^{-4}$	1·63 $\times 10^{-4}$	1·80 $\times 10^{-4}$	2·04 $\times 10^{-4}$	2½
3	7·00 $\times 10^{-7}$	1·70 $\times 10^{-6}$	3·62 $\times 10^{-6}$	5·17 $\times 10^{-6}$	6·75 $\times 10^{-6}$	8·61 $\times 10^{-6}$	9·62 $\times 10^{-6}$	1·10 $\times 10^{-5}$	3
3½	2·50 $\times 10^{-8}$	5·77 $\times 10^{-8}$	1·16 $\times 10^{-7}$	1·65 $\times 10^{-7}$	2·18 $\times 10^{-7}$	2·82 $\times 10^{-7}$	3·18 $\times 10^{-7}$	3·72 $\times 10^{-7}$	3½
4	5·41 $\times 10^{-10}$	1·19 $\times 10^{-9}$	2·31 $\times 10^{-9}$	3·28 $\times 10^{-9}$	4·34 $\times 10^{-8}$	5·68 $\times 10^{-9}$	6·48 $\times 10^{-9}$	7·71 $\times 10^{-9}$	4
4½	7·13 $\times 10^{-12}$	1·51 $\times 10^{-11}$	2·83 $\times 10^{-11}$	4·01 $\times 10^{-11}$	5·33 $\times 10^{-11}$	7·05 $\times 10^{-11}$	8·12 $\times 10^{-11}$	9·83 $\times 10^{-11}$	4½
5	5·72 $\times 10^{-14}$	1·17 $\times 10^{-13}$	2·13 $\times 10^{-13}$	3·02 $\times 10^{-13}$	4·03 $\times 10^{-13}$	5·38 $\times 10^{-13}$	6·25 $\times 10^{-13}$	7·69 $\times 10^{-13}$	5

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1.5. *A more general problem.* Takács[5] has considered a situation like Special Case 1 where a sequence of jobs, whose S-values are random variables, is fed to the production line. (The *S-value* of a job is the time needed to complete it, in the absence of stoppages.) Let $K_n(.)$ be the cumulative distribution function of the sum of the S-values of n consecutive jobs, which are about to be carried out next. Let $n(T)$ be the number of these jobs done during a total operating period of length T. Takács showed that the probability that $n(T)$ is less than n is

$$P(n(T)<n) = 1-\int_0^T K_n(x)\, d_x P(x,T),$$

where $P(x,T)$ is the off-schedule probability function discussed above. As is obvious intuitively, this expression for $n = 1$ reduces to the off-schedule probability $P(S,T)$ when $K_1(x) = 0$ for $x < S$ and 1 for $x \geqslant S$, which is the case where the first job has fixed length S. In general, if $K_n(x)$ is hard to evaluate directly, it can often be calculated by inverting numerically the nth power of the Laplace transform of $K(.)$.

2. THE SECOND RELIABILITY MODEL

2.1. *Specification.* Let the required operation have m stages. Let stage i be such that P_i is the probability that an error occurs during the working of stage i and is detected, by means of checks, before its completion. Let P_i^* be the probability that an error occurs during that stage but is *not* detected by the checks. Let $P_i + P_i' < 1$. Usually, $P_i^* \ll P_i \ll 1$. Use the following procedure for stage i:

(1) If $i = 1$, prepare to start stage 1. If $1 < i \leqslant m$, assume that stage $(i-1)$ has been completed and that no error has been detected in it. Prepare to start stage i.

(2) Carry out the operations of stage i.

(3) Check stage i. If the check tests are all passed, proceed to stage $(i+1)$ if $i < m$, and output the results or products if $i = m$. If the check tests are not all passed, proceed to (4).

(4) Stop the machine or production line. Find and repair the faults.

(5) Proceed to stage i, which must now be attempted again.

When the results are output from stage m, let them pass through a final check stage $(m+1)$, after stage m, which finds previously undetected faults with probability $\Pi(\leqslant 1)$. If no faults are found, the results are output; otherwise, the necessary locations and repairs of fault(s) are made, and then the sequence of operations, as described above, must be started again.

Let all the probabilities of error occurrence be mutually independent, both in different stages and in repetitions of the same stage.

Let $F_i(.)$ be the cumulative distribution function of the time v_i taken to carry out stage i and its checks. Let $R_i(.)$ be the cumulative distribution function of the time w_i taken to find and repair the fault(s) of stage i if any are detected. Let these cumulative distribution functions be independent of the number of attempts at stage i that have already been carried out. Let the times v_i, w_i all be mutually independent random variables, both in different stages and in repetitions of the same stage ($i = 1$ to $(m+1)$).

Let T_i and R_i ($i = 1$ to $(m+1)$) be positive numbers. Then *Special Case A* is the example of the second reliability model where

$$F_i(t) = 0 \text{ if } 0 \leqslant t < T_i;\ 1 \text{ if } t \geqslant T_i \quad (i = 1 \text{ to } (m+1)),$$

and

$$R_i(t) = 1 - e^{-R_i t} (t \geqslant 0) \quad (i = 1 \text{ to } (m+1)).$$

2.2. *Calculation of the off-schedule probability for the general case.* Let $G_i(.)$ be the cumulative distribution function of the time u_i taken to carry out stage i, allowing for possible repetition(s), before it is passed on to the next stage.

Let

$$Y_i(p) = \int_0^\infty e^{-pt}\, dG_i(t) \quad (i = 1 \text{ to } m)$$

$$\Phi_i(p) = \int_0^\infty e^{-pt}\, dF_i(t) \quad (i = 1 \text{ to } (m+1))$$

$$P_i(p) = \int_0^\infty e^{-pt}\, dR_i(t) \quad (i = 1 \text{ to } (m+1))$$

Theorem 7. (a) For $i = 1$ to m,

$$Y_i(p) = \frac{(1-P_i)\,\Phi_i(p)}{1 - P_i\,\Phi_i(p)\,P_i(p)}.$$

(b) The probability that an undetected error passes through stage i, after the necessary repetitions of stage i have been completed, is

$$P_i^{**} = P_i^*/(1-P_i) < 1.$$

Let the whole production procedure consist of stages 1 to m, together with a final check stage; let it take time v to carry out; then

$$v = \sum_{r=1}^{m} u_r + v_{m+1}.$$

Let the whole procedure be repeated if the final checks reveal error(s). Let u be the time needed to complete the whole production operation, allowing for possible repetition(s) of the whole procedure. Let v and u have respective cumulative distribution functions $F(.)$ and $G(.)$.

Let $$\Phi(p) = \int_0^\infty e^{-pt}\,dF(t), \qquad Y(p) = \int_0^\infty e^{-pt}\,dG(t),$$

$$P = 1 - \prod_{i=1}^{m} (1-P_i-P_i^*)(1-P_i)^{-1}.$$

Theorem 8.

(a) $$\Phi(p) = \left[\prod_{i=1}^{m} Y_i(p)\right]\Phi_{m+1}(p)$$

(b) $$Y(p) = \frac{(1-P\Pi)\,\Phi(p)}{[1-P\Pi\,\Phi(p)\,P_{m+1}(p)]}$$

(c) The probability that the procedure is finally completed incorrectly is $P^* = P(1-\Pi)/(1-P\Pi) < 1$. This probability is independent of the times taken to carry out the individual stages.

Let the symbol "$K_j(.)$" denote "jth cumulant of".

Theorem 9. (a) For $i = 1$ to m,

$$K_1(u_i) = (1-P_i)^{-1}\,[K_1(v_i) + P_i\,K_1(w_i)],$$

$$K_2(u_i) = (1-P_i)^{-1}\,[K_2(v_i) + P_i\,K_2(w_i)] + P_i(1-P_i)^{-2}\,[K_1(v_i) + K_1(w_i)]^2.$$

Mayne[1] gives a general formula for $K_j(u_i)\,(j \geqslant 1)$ in terms of the cumulants of v_i and w_i, and also gives explicit formulae for $j = 1(1)6$.

(b) For $$j \geqslant 1, \quad K_j(v) = \sum_{i=1}^{m} K_j(u_i) + K_j(v_m).$$

(c) $$K_1(u) = (1-P\Pi)^{-1}\,[K_1(v) + P\Pi\,K_1(w_{m+1})],$$

$$K_2(u) = (1-P\Pi)^{-1}\,[K_2(v) + P\Pi\,K_2(w_{m+1})] + P\Pi(1-P\Pi)^{-2}\,[K_1(v) + K_1(w_{m+1})]^2.$$

Mayne[1] gives a general formula for $K_j(u)\,(j \geqslant 1)$ in terms of the cumulants of v and w_{m+1} (and thus, by (a) and (b), in terms of the cumulants of v_i and w_i $(i = 1 \text{ to } (m+1))$), and also gives explicit formulae for $j = 1(1)6$.

(d) The probability of failing to complete a job within time T is $[1-G(T)]$. It can be estimated approximately, by means of the Cornish–Fisher[3] formula or one of its modifications, in terms of the cumulants of u, which are evaluated by (a), (b) and (c).

2.3. *Calculation of the off-schedule probability for special case A*

Theorem 10. If Special Case A holds, then

(a) $$\Phi_i(p) = e^{-pT_i} (i = 1 \text{ to } (m+1)).$$

(b) $$P_i(p) = R_i/(p+R_i) \quad (i = 1 \text{ to } (m+1)).$$

(c) $$Y_i(p) = \frac{(1-P_i)(p+R_i)\,e^{-pT_i}}{[p+R_i-P_i R_i e^{-p(T_i+R_i)}]} \quad (i = 1 \text{ to } m).$$

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(d) $Y(p)$ can be evaluated by applying parts (a), (b) and (c) of this theorem, together with Theorem 9a, to Theorem 9b.

Theorem 11. If Special Case A holds, then

$$K_1(u) = (1-P\Pi)^{-1}\left[\sum_{i=1}^{m}(1-P_i)^{-1}(T_i+P_i R_i^{-1}) + T_{m+1} + P\Pi\, R_{m+1}^{-1}\right]$$

and

$$K_2(u) = (1-P\Pi)^{-1}\left[\sum_{i=1}^{m} P_i(2-P_i)(1-P_i)^{-2} + P\Pi\, R_{m+1}^{-2}\right] + \left[K_1(u) + R_{m+1}^{-1}\right]^2.$$

In general, calculate the cumulants of u by using the results of Theorems 9 and 10.

3. APPLICATION TO PRODUCTION LINES, INCLUDING ELECTRONIC COMPUTERS AND DATA-PROCESSING SYSTEMS

3.1. *General principles.* The two reliability models discussed above were formulated because they seemed to be plausible representations, at least to a reasonably close first approximation, of a wide variety of production and processing operations, for example the operation of electronic computers and data-processing systems. *These models have not yet been verified adequately in practice*; in Mayne[2], their applicability to particular computers will be discussed.

The results for the first two computers considered are described briefly in Section 3.2. Unfortunately, they show that modifications to the models *are* sometimes needed in practice, but it is not yet known how far the behaviour of the two particular computers studied is representative of computers in general.

Thus most computers may in fact operate nearly according to the first reliability model.

In applying these models, exclude from consideration periods when the system is scheduled to be not running *or* is undergoing periodical routine maintenance *or* is being modified. These periods are essentially fixed and may be allowed for in advance while planning schedules. All other periods constitute the time potentially available for productive operation.

"Good" time consists of periods of operation without machine faults; it is assumed here that the process is sufficiently routine for other types of fault (in programming, design, etc.) to have been eliminated.

"Bad" time consists of time lost because parts of the production operation must be reworked owing to faulty operation of equipment, time taken to locate faults, and time taken to repair faults.

The second type of model is more realistic, but the first type is simpler to apply. If time lost due to reworking of production operations is only a small proportion of total "bad" time, then the two models will probably give answers closely agreeing with each other. The extent of such reworking times can be greatly reduced by splitting the production operation into small stages, each of which is checked for errors on its completion.

In theory, further improvements can be made to the models considered. For example, the performances of individual parts of the equipment can be taken into account, and possible interdependences between the performances of these parts can be allowed for. Very little evidence is yet available as to how far refinements of this sort will be needed in order to make the models of practical value.

While the two models discussed here are reasonably easy to handle mathematically, and are unlikely to involve calculations outside the range of an electronic computer, the calculations resulting from more complicated models may well be much more intractable and have to be carried out with the aid of Monte Carlo (simulation) methods.

Finally, it should be pointed out that the practical application of a model requires a knowledge of the parameters entering it (for example, the values of a and b in Special Case 2 of Model 1). Such parameters must usually be estimated empirically, and thus *allowance must be made for sampling errors when estimating the off-schedule probability*. Mayne[1] gives an outline of the methods needed to carry out such a modification. In practice, observations should probably be carried out on the equipment for several months at least, in order to reduce sampling errors to a reasonably small size.

3.2. *The study of a particular computer*. An attempt has been made to apply Special Case 2 of Model 1 to the operation of an actual computing system; for details, see Mayne[2]. Unfortunately, it was not possible to apply Special Case 2 outright, as its assumptions were found to be false. But a modification was proposed, which is thought to be reasonably plausible and to err, if anything,

on the side of safety. This modification allowed calculations to be carried out which were formally the same as those of Special Case 2.

An attempt was also made to apply Special Case 2 of Model 1 to the operation of a second computer system, with somewhat better results. This model fitted in all respects, *except* that the distribution of the lengths of "good" periods fluctuated from one week to another. Here, calculations of off-schedule probability were based on the *worst* weeks' performances, in order to be on the safe side.

For the computer systems studied, several values of the off-schedule probability were estimated numerically, and allowances to be made for sampling errors were assessed roughly.

It is hoped eventually to test the reliability models adequately, by considering the behaviour of a fairly large number of individual computers; the results of this survey will be summarized in Mayne[2]. The performance records of two further computers are already being investigated. The data so far examined suggest that extremely long "bad" periods, though still rare, occur much more often than would be the case if the "bad" time distribution were negative exponential.

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4. THE OPTIMUM ASSIGNMENT OF THE LENGTH OF THE SCHEDULED OPERATIONS

A balance needs to be found between too much caution (putting on too short a sequence of operations and letting the machine or system be idle for too long) and too tight a schedule (running too great a risk of not completing the operations on time).

The simplest model of this situation is as follows. Let $G(t)$ be the gain resulting if the operations are carried out in time t. It is required to *maximize the expected gain*, that is, to choose S so as to maximize

$$E[G(t)] = \int_S^\infty G(t)\, d_t H(S, t-S) = \int_0^\infty G(S+u)\, d_u H(S, u).$$

Let PS be the profit resulting from the completion on time of a sequence of operations of total length S. Let QS be the loss resulting from failure to complete this schedule on time; usually Q is much larger than P. Let C be the cost of a unit of production time. Assume that, if a schedule is not completed on time, it is not continued beyond time T. Then

$$G(t) = PS - Ct, \quad \text{if} \quad t \leqslant T; \quad -QS - ct, \quad \text{if} \quad t > T.$$

A more general model is as follows. Let U be the "ordinary" working period; obviously, $U \leqslant T$. Let C be the cost of a unit of production time during this period and $C(v)$ be the cost of working "overtime" for a period of length v. If $t \leqslant U$, work for the remainder of the "ordinary" period on a subsidiary project

yielding profit $(U-t)P'$, where $P'<P$. If $t \geqslant U$, work the required amount $(t-U)$ of overtime. Let the profit resulting from completion of the schedule be PS, for *any* value of t. If $t \geqslant T$, let there also be an off-schedule loss equal to $Q(w)$, where $w = t-T$. Then

$$G(t) = PS+(U-t)P'-CU, \quad \text{if} \quad t \leqslant U;$$

$$PS-CU-C(t-U), \quad \text{if} \quad U \leqslant t \leqslant T;$$

$$PS-Q(t-T)-CU-C(t-U), \quad \text{if} \quad t \geqslant T.$$

In practice, $C(v)$ might, for example, equal Dv, for some constant $D>C$, and $Q(w)$ might, for example, have form Qw^2, for some constant Q, or might alternatively equal some constant K if $w>0$; note that it is necessary, for consistency, that $C(0) = 0 = Q(0)$.

While no explicit analytic solution is available for *these* problems, approximate numerical calculation of the optimum value of S, based on numerical calculation of $H(S, u)$, using the results of Section 1, is fairly straightforward and should be reasonably easy to carry out with the aid of an electronic computer.

It may be added that there is considerable doubt as to what is the most suitable model for this situation, owing to the difficulty of formulating precisely the true objectives of the organization for which the schedule has to be carried out. The first possibility described above might occur if a customer refuses to buy any further products from a manufacturer who has allowed one of his orders to be fulfilled too late. The second possibility described above assumes (when $Q(.)$ varies continuously and is not a step function) that the loss of goodwill, as a result of late delivery, is more gradual.

5. OPTIMUM ASSIGNMENT OF WORK TO SUB-PERIODS OF A SCHEDULE

Suppose that a schedule is split into m sub-schedules (for example, a week into 5 working days). What is the best way of arranging the sub-schedules, so as to maximize the expected gain, as defined in Section 4? No detailed results have been worked out as yet, but the general principles of the procedure are stated, being basically an application of dynamic programming.

If $m = 2$:

First method. Preassign the length of working on Part 1. Work on Part 2 as long as necessary.

Second method. Work on Part 1 till a preassigned section of schedule is finished. Work on Part 2 as long as necessary.

Third method. Preassign the length of working on Part 1. Use this length of working if a preassigned section of the schedule has been finished by then. Otherwise, continue till the preassigned section of the schedule has been finished. Work on Part 2 as long as necessary.

If $m > 2$*:*

Each of the three methods for $m = 2$ can be generalized. For Part 1, proceed as before. For Part 2, proceed as for Part 1 of the remaining $(m-1)$-part problem. . . . For Part $(m-1)$, proceed as for Part 1 of the remaining 2-part problem. Work on Part m as long as necessary.

6. FURTHER PROBLEMS

Two generalizations of the problems described above are mentioned here, but no theoretical results have yet been derived for them.

Firstly, situations may arise where reserve capacity is desirable; in this case, find the best number of machines to use, balancing the cost of storing machines that are incompletely used against the loss resulting from failure to complete a schedule on time owing to shortage of machines.

Secondly, consider operations with several phases, each phase requiring several machines, each machine dealing with one phase only. Rather complicated queueing problems arise in this situation!

In addition to these problems, there is also the basic problem of considering whether the reliability models considered here are adequate for the particular production system under consideration, and, if not, how these models can most usefully be generalized. Much further practical and theoretical investigation is needed, on many types of equipment, before this problem can be considered to be adequately solved.

Problems of formulating objectives arise when optimum assignment of schedules is required. Probably, different models and objectives are appropriate to different fields of application.

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APPENDIX I

NOTE ON THE NUMERICAL INVERSION OF LAPLACE TRANSFORMS

Let $$\Phi(p) = \int_0^\infty e^{-pu} f(u)\, du.$$

Suppose that it is required to evaluate $f(u)$.

A method which is sometimes very useful for calculating $f(u)$ approximately for large u is as follows. Suppose that

$$\Phi(p) = a(p)/b(p),$$

where $a(p)$ is analytic,

$$b(p) = \prod_{i=1}^{\infty} (p - c_i)^{-1},$$

where $a(s)$ and $b(s)$ have no roots in common, and all the c_i are distinct. Then

$$f(u) = 0 (u < 0), \quad \sum_{i=1}^{\infty} \frac{a(c_i)}{b'(c_i)} e^{-c_i u} (u \geqslant 0),$$

where $b'(.)$ is the first derivative of $b(.)$.

For other methods of inversion, see, for example:

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APPENDIX II

NOTE ON AN APPROXIMATE METHOD FOR EVALUATING THE OFF-SCHEDULE PROBABILITY

Let $x_i = K_i(S)/K_1(S)$, $y_i = K_i K_1^{-i}$, $z_i = K_i(u) K_1^{-i}/K_1(S)$, $q = z_2^{-\frac{1}{2}}$, $w_i = z_i q^i / i!$, using the notation of Section 1.2.

Then, applying Theorem 2, it is found that

$$z_1 = 1, \quad z_2 = x_2 + y_2, \quad z_3 = x_3 + 3x_2 y_2 + y_3,$$

$$z_4 = x_4 + 6x_3 y_2 + x_2(4y_3 + 3y_2^2) + y_4,$$

$$z_5 = x_5 + 10x_4 y_2 + 5x_3(2y_3 + 3y_2^2) + 5x_2(y_4 + 2y_2 y_3) + y_5,$$

$$z_6 = x_6 + 15x_5 y_2 + 5x_4(4y_3 + 9y_2^2) + 15x_3(y_4 + 4y_2 y_3 + y_2^3) + x_2(6y_5 + 15y_2 y_4 + 10y_2^2) + y_6, \quad \text{etc.}$$

Let $\quad c = [K_1(S)]^{\frac{1}{2}}, \quad d = (T-S)^{\frac{1}{2}} K_1^{-\frac{1}{2}}, \quad k = d - c, \quad t = 2qk,$

$$W_1 = w_3 + (1 - 4w_3 q) q k^2,$$

$$W_2 = 2(3w_4 - 7w^2) qk - 4[w_3 + 2(w_4 - 4w_3^2) q] q^2 k^3,$$

$$W_3 = -(3w_5 - 15w_3 w_4 + 17\tfrac{1}{3} w_3^3) + [3w_4 - 7w_3^2 + 4(6w_5 - 42w_3 w_4 + 62\tfrac{1}{3} w_3^3) q] qk^2 - [w_3 + 12(w_4 - 4w_3^2) q + 16(w_5 - 11w_3 w_4 + 23w_3^3) q^2] q^2 k^4,$$

$$W_4 = -[30w_6 - 204w_3 w_5 - 105w_4^2 + 912w_3^2 w_4 - 824\tfrac{1}{3} w_3^4] qk + \begin{bmatrix} 4(6w_5 - 42w_3 w_4 + 62\tfrac{1}{3} w_3^3) + \\ (80w_6 - 768w_3 w_5 - 384w_4^2 + 4{,}376w_3^2 w_4 - 4{,}837\tfrac{1}{3} w_3^4) q \end{bmatrix} q^2 k^3 - \begin{bmatrix} 6(w_4 - 4w_3^2) + 32(w_5 - 11w_3 w_4 + 23w_3^3) q + \\ 16(2w_6 - 28w_3 w_5 - 15w_4^2 + 222w_3^2 w_4 - 316w_3^4) q^2 \end{bmatrix} q^3 k^5,$$

$$t_1 = t + W_1 c^{-1}, \quad t_2 = t + W_1^2 (W_1 c - W_2)^{-1},$$

$$t_3 = t_1 + W_2^2 c^{-1} (W_2 c - W_3)^{-1},$$

$$t_4 = t_2 - \frac{(W_2^2 - W_1 W_3)^2 (W_1 c - W_2)^{-1}}{[(W_2^2 - W_1 W_3) c^2 + (W_1 W_4 - W_2 W_3) c + W_3^2 - W_2 W_4]}.$$

Then $[1 - \Phi(t)]$, $[1 - \Phi(t_i)]$ ($i = 1$ to 4) are successive approximations to $P(S, T)$, $\Phi(.)$ being the standard normal cumulative distribution function. For the derivation of this method, see Mayne[1].

For Special Case 2 of Model 1, c, d, k, are as defined in Section 1.4, $g = 2^{-\frac{1}{2}}$, $w_i = 2^{-\frac{1}{2}i}$. This yields the approximations quoted in Section 1.4; in fact, $t_2 = 2^{\frac{1}{2}} k'$, $t_3 = 2^{\frac{1}{2}} k^*$, $t_4 = 2^{\frac{1}{2}} k^{**}$. This suggests that, for the general case of Model 1, $[1 - \Phi(t_2)]$ could be used as a reasonably good first approximation to $P(S, T)$, while $[1 - \Phi(t_3)]$ and $[1 - \Phi(t_4)]$ should be used for more accurate work.

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1. In his book on *Documentation* (1948) the librarian, S. C. Bradford[1], discussed certain regularities in the pattern of distribution of articles on a particular subject over different journals. He gives as examples the distributions for geophysics from 1928–31 inclusive and lubrication for 1931 to June, 1933, inclusive. The typical picture is one in which, to a bibliography covering a certain short term of years, a few journals contribute a large number of articles, more journals contribute fewer and so on in a monotonic sequence ending with a large number of journals contributing one article each. Patterns of this kind have been observed by several authors, but Bradford was the first, I think, to advance an explanation of the effect in bibliographical terms.

2. Table 1 shows an analysis of the recent bibliography of operational research published by ORSA[4] in 1958. I have excluded books and unpublished or classified reports and thus include only articles actually published in available journals. For convenience of comparison the columns are arranged in the same form as those given by Bradford.

3. Without any more penetrating analysis we can at once draw some important conclusions from this table. The first five journals cover 611 out of 1763 references, about one-third. If we want a 50 per cent cover we need 18 journals.* A 75 per cent cover requires 67 journals. There is a rapidly diminishing return by way of additional articles per journal as we proceed down the table. The implications for bibliographical work in operational research are clear. Assuming that all the articles are of equal standing (in the sense that it may be necessary to consult any one to see whether it is relevant to some particular inquiry), we might expect a normal operational research library to contain the 18 journals required for a 50 per cent cover. To go much further would obviously strain its resources in money and space.

4. For a young subject like operational research the pattern may of course change; but the width of the domain which it covers suggests that the extent of the scatter of the literature will not diminish. In this connection, a similar pattern for an older subject, statistics, may be of some interest. Table 2 shows a classification of 1465 references to statistical methodology (covering the period 1925–39) given at the end of Volume 2 of my *Advanced Theory of*

* The 18 journals were as follows: *Operations Research, N.R.L.Q., Operational Research Quarterly, Management Science, Econometrica, Journal of Industrial Engineering, Journal of the Royal Statistical Society, Annals of Mathematical Statistics, Journal of the American Statistical Association, Industrial Quality Control, Applied Statistics, Proceedings of the National Academy of Sciences, Journal of Farm Economics, Journal of the Iron and Steel Institute, Operations Research Digest, Journal of the Society for Industrial and Applied Mathematics, Factory Management and Maintenance, Advanced Management.*

Statistics.[2] This is highly selected but the characteristic pattern has survived. For some further examples see Simon (1955).[5]

5. There is an obvious resemblance between this type of distribution and that of income. If we equate journals to persons and number of references to size of income we have the characteristic pattern in which a lot of people (journals) have a low income (few references) proceeding regularly to a few millionaires (journals with a large number of references). The statistician's method of analysing Table 1, were it an income distribution, would be to

TABLE 1. 1763 REFERENCES TO OPERATIONAL RESEARCH

A No. of journals x	B No. of references $g(x)$	C Cumulative Σx	D Cumulative $\Sigma x g(x)$	E log col. C
1	242	1	242	0·000
1	114	2	356	0·301
1	102	3	458	0·477
1	95	4	553	0·602
1	58	5	611	0·699
1	49	6	660	0·778
1	34	7	694	0·845
2	22	9	738	0·954
2	21	11	780	1·041
2	20	13	820	1·114
1	18	14	838	1·146
4	16	18	902	1·255
2	15	20	932	1·301
1	14	21	946	1·322
2	12	23	970	1·362
5	11	28	1025	1·447
3	10	31	1055	1·491
4	9	35	1091	1·544
8	8	43	1155	1·634
8	7	51	1211	1·708
6	6	57	1247	1·756
10	5	67	1297	1·826
17	4	84	1365	1·924
29	3	113	1452	2·053
54	2	167	1560	2·223
203	1	370	1763	2·568

cumulate *from the bottom*, i.e. starting at the low incomes. He would then graph the sum $\Sigma x f(x)$ cumulated from the bottom as ordinate against the sum Σx cumulated from the bottom as abscissa to obtain Gini's curve of concentration or the Lorenz curve (Kendall and Stuart, 1958, **1**, p. 48).[3]

6. Bradford, however, cumulated from the other end and produced figures analogous to those of columns C and D in Tables 1 and 2. He then remarked that if we put C on a logarithmic scale, as given in column E, and graph D

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as ordinate against E as abscissa, the result is nearly linear over a large part of the range.

Figure 1 shows such a graph for the data of Table 1. The linearity is remarkable and in fact is better than in Bradford's own examples.

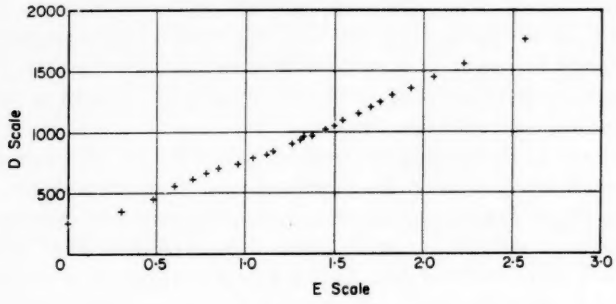

FIG. 1.

TABLE 2. 1465 REFERENCES TO STATISTICAL METHODOLOGY

A No. of journals x	B No. of references $g(x)$	C Cumulative Σx	D Cumulative $\Sigma xg(x)$	E log col. C
1	273	1	273	0·000
1	212	2	485	0·301
2	66	4	617	0·602
1	60	5	677	0·699
2	59	7	795	0·845
1	58	8	853	0·903
1	49	9	902	0·954
1	46	10	948	1·000
2	36	12	1020	1·079
1	28	13	1048	1·114
1	20	14	1068	1·146
1	18	15	1086	1·176
3	17	18	1137	1·255
1	16	19	1153	1·279
2	13	21	1179	1·322
4	10	25	1219	1·398
1	9	26	1228	1·415
1	8	27	1236	1·431
1	7	28	1243	1·447
3	6	31	1261	1·491
2	5	33	1271	1·519
5	4	38	1291	1·580
12	3	50	1327	1·699
24	2	74	1375	1·869
90	1	164	1465	2·215

7. It is therefore very tempting to seek a model which will explain the Bradford effect. Bradford advanced a hypothesis of the following kind: there are certain journals which are specifically devoted to the subject under examination, and these form a nucleus. Around this nucleus we may picture zones, each containing the same number of articles as the nucleus itself. The first zone is, so to speak, occupied by journals which are partly relevant, and contains, say, k times as many journals as the nucleus. The second zone consists of less relevant journals, standing to the first zone in much the same relation as the first zone to the nucleus; we may therefore suppose that it contains k^2 as many journals as the nucleus. And so on for further zones.

8. On the basis of this picture of periodicals as a family of successive generations of diminishing kinship Bradford thought that he had arrived at an explanation of his linear law. His demonstration, however, involved an oversight which was pointed out and corrected by Vickery (1948)[6] who showed that on Bradford's model the theoretical graph of D against E was a gentle curve.

9. There is a completely different way of looking at this topic which subsumes the Bradford-type distribution under a general class of distribution named by Simon (1955)[5] the Yule distribution. I have remarked in paragraph 5 that the more usual way of considering the data of Table 1 would be to start at the other end thus:

TABLE 3

A_1 No. of references i	B_1 No. of journals $f(i)$	C_1 No. of journals containing i or more references	D_1 Column A_1 times Col. C_1
1	203	370	370
2	54	167	334
3	29	113	339
4	17	84	332
5	10	67	335
6	6	57	342
7	8	51	357
8	8	43	344
9	4	35	318
10	3	31	310
..	..	..	..
..	..	..	..
..	..	..	..
..	..	..	..
102	1	3	306
114	1	2	224
242	1	1	242

If we multiply the number of references i (col. A_1) by the number of journals containing that number or more (col. C_1), we get the remarkably constant

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figures of column D_1. In short, this is almost a case of what is known as Zipf's law (Zipf, 1949).[7] Simon (1955)[5] states this in the form: for the U.S.A. and most Western countries in the present century, if $F(i)$ is the number of cities of population greater than i, then

$$F(i) = \frac{A}{i^{\rho}} \tag{1}$$

where ρ is very nearly unity and A is some constant.

10. This remarkable law is approximately a particular case of one which Simon derives in the form

$$f(i) = AB(i, \rho+1) \tag{2}$$

where, in our present context, $f(i)$ is the frequency of journals containing i references and B is the Beta function. Since, for large n,

$$\frac{\Gamma(i)}{\Gamma(i+n)} \sim i^{-n}, \tag{3}$$

equation (2) can be written as

$$f(i) \sim \frac{\text{constant}}{i^{\rho+1}} \tag{4}$$

and a summation gives us approximately equation (1).

11. Simon derives (2) as the steady state solution of a stochastic process. In our present terms his basic assumptions can be stated thus:

(1) If p articles have appeared, the probability that a new $(p+1)$th article will appear in a journal which already has i references is proportional to $if(i)$, that is to the total number of references in journals containing i references each. The pattern of Table 1, in fact, tends to perpetuate itself.

(2) But there is also a probability that the $(p+1)$th article may appear in a journal not hitherto included in the table. This is a constant, say equal to $1-\rho^{-1}$.

On these assumptions, equation (2) may be derived. They may, of course, be relaxed to give more general results. It appears to me that such a model is more plausible than Bradford's.

12. It remains to be examined why Bradford's way of looking at a Yule distribution with $\rho = 1$ gives an approximation to a straight line. The reason, I think, is as follows:

Column C of Table 1 is the inverted form of column C_1 and Σx of column C is thus the sum of $f(i)$ from the bottom of the table. Column D of Table 1 is the sum of $if(i)$ from the bottom in columns A_1 and B_1. Thus for Bradford's linear law to be true, we require that

$$\log \Sigma f(i) = \text{linear function of } \Sigma if(i),$$

the summation being from the bottom in Table 3.

If N is the total number of references (1763 in our case), M the number of journals, this is equivalent to

$$\log\left(N-\sum_{j=1}^{i} f(j)\right) = \text{l.f.}\left(M-\sum_{j=1}^{i} jf(j)\right) \tag{5}$$

or, if $f(i)$ refers to *relative* frequency,

$$\log\left\{1-\sum_{j=1}^{i} f(j)\right\} = \text{l.f.}\left\{\sum_{1}^{i} jf(j)\right\} \tag{6}$$

Now suppose that

$$\sum_{j=1}^{i} f(j) = 1-\frac{1}{i+1}. \tag{7}$$

This entails that the frequency of journals containing more than i references is $1/(1+i)$, not $1/i$ as in Zipf's law. The form (7), however, is of the Yule type, as discussed by Simon. Then the left-hand side of (6) becomes $-\log(1+i)$. Further,

$$f(i) = \frac{1}{i(i+1)}$$

and hence

$$\sum_{1}^{i} jf(j) = \sum_{1}^{i} \frac{1}{j+1} \simeq \log(1+i),$$

and thus the equivalence (6) is established.

13. In the present state of development it does not seem worthwhile pursuing these theoretical results much further. But when the subject has settled down it might be worthwhile reviewing the distributions to see whether a better fit is obtained with $\rho > 1$. To come back for a moment to the matter of bibliography, it must be remembered that the problem of the outliers is not completely described in terms of existing bibliographies. There is also a non-observed class of journals which have not carried a relevant article in the period examined but may do so at any moment in the future. One would like to be able to estimate the size of this potentially contributory class.

ACKNOWLEDGEMENTS

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A Method for Finding the Shortest Route through a Road Network

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When a new road is being planned it is necessary to assess how much traffic will be diverted to it from various parts of the existing road network. This allocation of traffic has usually been based on a comparison of journey times or journey costs on alternative routes, and has depended on the selection, by trial and error, of the cheapest route through the network.

A method is described which determines the shortest or cheapest routes between points on the network and which can readily be extended to show how traffic between the points is distributed and to assess the total cost of vehicle operation on the network.

The procedure is quite systematic and independent of the manner in which journey costs are derived, and it could be carried out on an electronic computer with considerable saving in time. It can be applied to any transportation or communication problem that involves finding the most economical routes through a network.

INTRODUCTION

WHEN a new road is being planned it is necessary to assess how much traffic will be diverted to it from various parts of the existing road network. This allocation of traffic has usually been based on a comparison of journey times or journey costs on alternative routes.[1,2] The comparison, for each origin–destination pair, should logically be made between the lowest journey cost under existing conditions and that expected when the new roads are built; the "cost" of a journey can, of course, be extended beyond purely financial units and may be expressed in terms of time or distance or some other measure of the attractiveness of the road. The cheapest route is generally determined by trial and error and since this comparison is made for traffic movements between every pair of points or zones in the study it can become a very time-consuming task. It is, of course, even more tedious if an attempt is made, by repeated approximations, to allow for the effect of changes in flow on the cost of travel. This effect will not, however, be considered here.

Dantzig[3] has suggested that, to determine the cheapest routes from a given origin, a plan of the road network should be taken and sufficient roads on it marked to form a "tree" whose branches reach or pass through every point of intersection on the network but do not form any closed loops, i.e. so that only one path exists from the origin to any other point in the network.

The cost of the journey along the tree from the origin to any point can easily be calculated, so it is possible to obtain the cost of the journeys to any two points which lie one at each end of a link of the road network not used for forming the tree. If the difference between these journey costs is greater than the cost of travel along the link under consideration then the tree is modified. The modification consists of inserting this link and removing the last link on the original journey to the more costly point of the two considered. This process is continued until none of the unused links can achieve a saving in travel cost, and it is evident that this method can be shortened if correct judgment is used in the selection of the original tree. A similar but more systematic method has been put forward by Bellman[4].

Moore[5] has proposed a solution in which distances to the points on the network are built up step by step outwards from the origin. All links from the point are considered in turn and combined with the journeys to this point from the origin to establish the distances from the origin of the new points. If such a calculation provides a shorter journey to a given point than one already available, the shorter journey is recorded. However, this method does not provide a rule as to the order in which the points are considered and considerable care must be exercised to ensure that the effect of any reduction in distance to a point is passed on to all adjacent points.

Berners-Lee[6] has suggested a solution for the best path on a regular lattice in which one's directions of travel are limited to (say) northwards and eastwards. Such a route is not necessarily the quickest route although he does state that the method can be extended to include southward and westward movements in addition.

A neat analogy by Minty[7] for determining the cheapest route through a network in which all roads allow two-way movement is based on a string net, in which the knots represent the road junctions and the length of string between each pair of knots is proportional to the cost of travel between the corresponding junctions. The cheapest path between any two points can be selected by pulling them apart as far as possible. The line of tight strings between them will be made up of those sections which form the cheapest route and its length will represent the cost of the journey. An extension of this analogy is that if weights are placed at each of the junctions and the net is suspended from one point then the depths at which the other weights hang below it will represent the cheapest routes between them and the point of suspension.

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METHOD

A method has been developed at the Road Research Laboratory to determine the cheapest routes between points on a road network, which can if necessary include one-way streets. The method can show how traffic between the points is distributed on the sections of road and can give the total cost of vehicle operation on the network. At each stage of the method the next step is automatic and there is no uncertainty as to when the procedure is complete.

The method is described below and to simplify the explanation it has been presented in terms of distance between points in the network rather than in terms of cost.

Suppose that we have a number of points joined by a network of links of various lengths. We choose one, P_0 say, as origin and wish to find the shortest distances from P_0 to the remaining points. The shortest distance between two points will here be referred to as the distance between them. Suppose the problem solved and that the points are listed in order of increasing distance from P_0 : P_1 at distance d_1, P_2 at distance d_2, P_3 at distance d_3 and so on, where $d_1 \leqslant d_2 \leqslant d_3 \leqslant \ldots$. Consider how we could find P_{r+1} if we had already found $P_1, P_2, \ldots P_r$ and their distances $d_1, d_2, \ldots\ldots d_r$ from P_0.

It can be seen that the shortest path to P_{r+1} must be by a single link from a point P_s, which must be one of the points $P_0, P_1, P_2, \ldots P_r$ because if it were not there would be at least one intermediate point (not one of $P_0, P_1, \ldots P_r$) which would be nearer to P_0 than P_{r+1} and this is impossible, since P_{r+1} is by definition the next nearest point. This link is also the shortest of the links emerging from the point P_s and going to a point whose shortest distance is not yet known, for if not, the point on the end of the shortest link would be nearer to P_s and therefore P_0 than P_{r+1} and this is impossible. If therefore $l_0, l_1, l_2, \ldots$ are the shortest of these links emerging from $P_0, P_1, P_2, \ldots$, we form the sums $l_0, l_1 + d_1, l_2 + d_2, \ldots l_r + d_r$ and the least of these indicates the link required to lead to P_{r+1}. In cases where there is no link to a point whose shortest distance is not yet known, then the sum $l + d$ is omitted. It is possible that two points will satisfy the requirements for P_{r+1}; their $l+d$ values being equal and less than any remaining value of $l+d$. If so either point may be taken as P_{r+1} and the method will then automatically select the other as P_{r+2}.

By starting from P_0 and following the procedure just outlined we can find P_1, P_2, etc., in succession. In the systematic method for carrying out this process we form, at each stage, the sums of the distances using all

the links to points whose shortest distances are not yet known because, although at any given stage only the shortest is required, the others may become the shortest at a later stage and therefore it is convenient to form all the sums immediately the point is reached.

This process can be performed graphically or it can be reduced to the making of systematic entries in a table. It caters equally well for the situation where the "distance" or cost of travel along a link varies according to the direction of movement; this includes the special case of a one-way street. Simple systematic extensions of the table show how traffic on the network is distributed and can be used to assess the total cost of vehicle operation on the network. Further details of this tabular method are given in Road Research Laboratory Research Note No. RN/3337, obtainable from the Director of Road Research, Road Research Laboratory, Harmondsworth, Middlesex.

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CONCLUSIONS

The method provides basic information on the use of a road network without requiring the exercise of any judgment apart from that necessary to define the original network. The procedure is quite systematic and it could readily be carried out on an electronic computer with considerable saving in time. It can, of course, be applied to any transportation or communication problem that involves finding the most economical route through a network.

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Dynamic Programming and Inventory Problems

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After an introductory discussion of the usefulness of the technique of dynamic programming in solving practical problems of multi-stage decision processes, the paper describes its application to inventory problems. In particular, the effect of allowing the number of decision stages to increase indefinitely is investigated, and it is shown that under certain realistic conditions this situation can be dealt with. It appears to be generally true that the average cost per period will converge, for an optimal policy, as the number of periods considered increases indefinitely, and that it is feasible to search for the policy which minimizes this long-term average cost. The paper concludes with a specific example, in which it is shown that only eight iterations were necessary to find a reasonable approximation to the optimal re-order policy.

THE concept of "dynamic programming" is comparatively recent; the fundamental principle on which the ideas used are based was first given in a published paper by Richard Bellman in 1952.* In eight years Bellman and others have developed the theory to the point where it must be considered part of the stock-in-trade of every operational research worker who has to concern himself with the mathematical structure of decision processes. In this paper we will present some of the basic ideas, a little of the underlying mathematics, and some applications to inventory problems. It is to be regretted that there are as yet few published case studies, and it is in the hope of stimulating such papers that this article is written.

Let us start by discussing the two classes of situations in which decisions are made. In the simplest case, we have to select a course of action once and only once. The decision to keep R.A.F. fighters in England during the battle of France was of this type. If the wrong choice had been made the results might have been so catastrophic that no further decisions would have been required. On the other hand, we are frequently faced with situations where we are called upon to make a series of decisions, each of which depends on the outcome of a previous action. Decisions about the fraction of reserve forces to be committed in battle are of this type. Each decision depends on the outcome of the battle resulting from the previous decision.

How can we make such decisions on a rational basis? It seems clear that each of them should take into account its effect on the future, and in practice this is done. Thus we do not maximize output one month by neglecting plant maintenance to the point where production is reduced in the next month.

* The best discussion of Bellman's work may be found in his book.[2]

The first difficulty in formulating models for recurrent decision processes is to decide on the number, n, of stages over which we wish to optimize. We will see later that in many situations we can avoid this problem because for large n the decision is virtually independent of n. In other words, the decision tends to a limiting form as the number of stages increases indefinitely. For the present we will assume that the process has a natural horizon and that n is given.

Suppose that initially the situation may be described by a set of state variables $s \equiv (s_1, s_2, \ldots)$ and that a decision consists of choosing a transformation T_i from a set of possible transformations $T \equiv (T_1, T_2, \ldots)$ such that the result of the decision is to yield a new set of state variable $s' \equiv T_i s$, about which the next decision must be made. In the deterministic case s' is specified exactly by T_i and s. In the probabalistic case s' may also depend on the values of some chance variables subject to probability distribution. We define a policy, P_n, as a set of rules for determining the transformation at each stage. Let $F_n(s, P_n)$ be the n stage return when the initial state is s and we use the policy P_n. In the deterministic case, F_n is completely defined; in the probabalistic case, F_n must be understood to be the expected return with respect to the chance variables.*

Our problem consists of choosing a policy P_n which will maximize the return. Such an optimum return can only depend on the number of stages and the initial state s, so that we may denote it by $f_n(s)$ and write

$$f_n(s) = \max_{P_n} F_n(s, P_n). \tag{1}$$

Instead of using equation (1) as it stands, let us consider the relationship between P_n and P_{n-1}. Any policy P_n for n stages must consist of an initial decision T plus a policy P_{n-1} for the remaining $(n-1)$ stages. Now T will result in an initial state Ts for the second decision,† so that the return from the last $(n-1)$ decisions will be $F_{n-1}(Ts, P_{n-1})$ and the total return will be some function ϕ of s, T and $F_{n-1}(Ts, P_{n-1})$, i.e.

$$F_n(s, P_n) = \phi[s, T, F_{n-1}(Ts, P_{n-1})]. \tag{2}$$

Since the payoff from any stage onwards is independent of all previous stages, if we know the policy P_{n-1} which maximizes $F_{n-1}(Ts, P_{n-1})$, we can find the optimal P_n by choosing a suitable T. But

$$\max_{P_{n-1}} F_{n-1}(Ts, P_{n-1}) = f_{n-1}(Ts),$$

so that

$$f_n(s) = \max_{P} F_n(s, P_n) = \max_{T} \phi[s, T, f_{n-1}(Ts)]. \tag{3}$$

* Other formulations are possible; we would, for example, define F_n as the probability of exceeding some fixed cost, but we will not do so here.

† It must be understood that in the probabilistic case we will consider expected values. In this case the right-hand side of equations (2) and (3) must be interpreted as the expected value of the expression shown.

Thus, if we know the $(n-1)$ stage optimal policy, we can find the n-stage policy by searching among the possible decisions at the first stage only. Equation (3) constitutes Bellman's "Principle of Optimality", which may be stated in words:

> "*An optimal policy has the property that whatever the initial state and decision may be, the remaining decisions must constitute an optimal policy with respect to the state resulting from the initial decision.*"*

In the one-stage problem we have

$$f_1(s) = \max_T F_1(s, T),$$

so that $f_1(s)$ may be found by searching among the possible decisions at one stage only. We may then proceed inductively to find $f_n(s)$. The advantage of this method is we have reduced our problem of searching among n-stage policies to a series of n problems in each of which our search is limited to the possible decisions at one stage only. Of course, we cannot tell in advance what set s will occur at each stage, so we have to find the optimal decisions for all possible s. However, practical experience will often suggest limitations on the regions of s to be considered beyond those required by the mathematical possibilities.

The principle of optimality has a wide range of application in many fields, including production scheduling, allocation of resources, sequential gaming, investment problems, and many others. However, we will limit ourselves to illustrations from inventory theory. In the simplest cases the set of state variables consists of the existing inventory level; at fixed periods, say once a month, we have to make a decision about the quantity to be purchased. We will use s to denote the initial inventory level and q to denote the quantity purchased. The costs during the month following the purchase decision will be a function of s, q and the customer demand. In general, customer demand will not be known until the month is over, so we can only base our decision on the expected cost with respect to demand.† The expected cost is thus some function of s and q, but it is convenient to write it as a function of the initial inventory s, and the quantity on order or in hand, $S = s+q$. We will use $I(s, S)$ for the expected costs during the month following the decision. Equation (3) now takes the form

$$f_n(s) = \min_{S \geqslant s} F_n(s, S) = \min_{S \geqslant s} \left\{ I(s, S) + \int_0^\infty f_{n-1}(S-x)\, dP(x) \right\} \qquad (4)$$

where $P(x)$ is the *distribution function* for demand x. [We assume delivery takes place during the month and unfilled orders result in a back-log to be filled later; other assumptions are possible and require suitable modifications.]

* For a discussion of this principle, see reference (5).

† Lead time and purchase price may also be chance variables, but we will not discuss them here.

We also have

$$f_1(s) = \min_{S \geqslant s} I(s, S). \tag{5}$$

We can solve (5) and proceed to solve (4) for successive values of n. If $I(s, S)$ is differentiable with respect to S, we may use calculus, but we must bear in mind the limitation $S \geqslant s$ resulting from our inability to order negative quantities. There may also be a limitation due to limited storage facilities. We have to examine $I(s, S)$ on all boundaries as well as at intermediate points. Unfortunately, there are some very common examples where $I(s, S)$ is not differentiable at all possible values of S. If there is a fixed set-up cost, k, associated with ordering any quantity, however small, then $I(s, s+\epsilon) - I(s, s) \to k$ as $\epsilon \to +0$; $I(s, S)$ is discontinuous with respect to S at $s = S$. Additionally, there may be price-breaks associated with the ordering of large quantities which introduce further discontinuities. In such cases we may be forced to use some form of sequential search for the minimizing S, until we have located it within narrow enough limits for our purposes. Stuart Dreyfus has discussed some possible methods in these circumstances.[4]

Of course, if $I(s, S)$ is discontinuous, we will have the same search problem at each iteration. The advantages of reducing the n-stage problem to n separate problems in one variable each, rather than one problem in n variables, should now be apparent.

We return now to the problem of determining the number of stages over which we wish to optimize. When there is no apparent limitation on n it is natural to inquire what happens as n is increased indefinitely. Is it possible that $f_n(s)$ could tend to a limit as n tends to infinity?

In general, $f_n(s)$ may be expected to diverge unless we specially introduce something in the structure of the problem which makes it do otherwise. For example, in equation (4) the cost over n periods is the sum of the costs in each separate period, and a necessary—but not sufficient—condition for convergence is that the costs in each period tend to zero. We may easily suppose that, in any case of practical importance, $I(s, S)$ is greater than some positive quantity for all S, and it follows that $f_n(s)$ will diverge. There are two ways of avoiding this divergence. In the first we note the fact that the payments occur at different points of time and instead of minimizing the total expected cost, we minimize the *total expected discounted cost*. We define $f_n(s)$ as the minimum total expected discounted cost and replace (4) with

$$f_n(s) = \min_{S \geqslant s} \left\{ I(s, S) + v \int_0^\infty f_{n-1}(S - x)\, dP(x) \right\} = \min_{S \geqslant s} F_n(s, S) \tag{4a}$$

where $F_n(s, S)$ is now a discounted cost; $v = 1/1+i$ is the discount factor for the period between decisions, and i is the corresponding rate of interest.

In (4a), $I(s, S)$ is considered to occur immediately and $f_{n-1}(S-x)$ occurs one period hence.

It is now possible to show that

$$|f_n(s)-f_{n-1}(s)| \leqslant v|f_{n-1}(s)-f_{n-2}(s)|$$

and hence

$$\sum_{r=2}^{n}\{f_r(s)-f_{r-1}(s)\}$$

converges to a limit $f(s)$.

Moreover, $f(s)$ is the unique solution to the functional equation

$$f(s) = \min_{S \geqslant s}\left\{I(s,S)+v\int_0^{\infty} f(S-x)\,dP(x)\right\}. \tag{6}$$

Because of the uniqueness of the solution of (6) it is not necessary to start with

$$f_1(s) = \min_{S \geqslant s} I(s,S)$$

in performing iterations. If, from practical experience or otherwise, we know an approximation to the optimal policy, we can find the corresponding discounted cost and commence our iterations with this value for $f_1(s)$. Depending on how good our approximation was, we may obtain much quicker convergence than is possible starting with $f_1(s)$. We may sometimes guess the structure of the solution, subject say to some unknown parameters, and immediately compute the parameters, thus verifying our guess.

The introduction of a discount factor usually solves all the mathematical problems of convergence, but it presents some practical difficulties when decisions are to be made fairly frequently. It is apparent that if decisions are to be made, say monthly, the discount factor must be very near to unity—perhaps differing from unity by less than the errors of measurement in some of the other parameters. In this case it is unreasonable to insist on introducing a discount factor merely to satisfy the mathematical representation we choose to use. Rather we must reformulate our mathematics. In order to see what happens when the discount factor tends to one, let us define $G_n(s,S)$ as the fixed monthly payment equivalent in present value to $F_n(s,S)$, i.e.

$$G_n(s,S) = \frac{1-v}{1-v^n}F_n(s,S).$$

Now any policy which minimizes $F_n(s,S)$ will also minimize $G_n(s,S)$, so we could just as well define an optimal policy as minimizing $G_n(s,S)$. However, for any fixed n, as $v \to 1$, $G_n(s,S)$ tends to $1/n$ times $\lim_{v\to 1} F_n(s,S)$, so that the limiting policy as $v \to 1$ will minimize the average cost per period for each n. Thus we might expect that the policy obtained by ignoring interest and putting $v-1$ will minimize the long-term average cost per period.

To sum up: If we ignore interest, then, for all policies, the total cost over n periods will tend to infinity as n increases indefinitely. However, it is generally true that the average cost per period will converge and it is quite feasible to search for the policy which minimizes the long-term average cost per period. Such a policy may be found iteratively by searching for policies which minimize total costs over 1, 2, 3, ... periods, using equation (4).

We conclude with an example. Suppose that $I(S)$ is the expected holding and shortage cost during a month which starts with inventory on hand or on order equal to S, and that it costs k to place an order for additional inventory.

We have

$$f(s) = \min_{S \geqslant s} \left\{ I(S) + g(S-s) + v \int_0^\infty f(S-x)\, dP(x) \right\}$$

where

$$g(u) = \begin{cases} k & u > 0 \\ 0 & u = 0 \end{cases}.$$

Unfortunately, no general solution to this equation is known. It is tempting to assume a solution of the form

$$\left.\begin{array}{lll} S = S_0 & \text{if} & s \leqslant s_0 \\ S = s & \text{if} & s > s_0 \end{array}\right\} \text{where } s_0 \text{ and } S_0 \text{ are suitable, chosen.}$$

This is the famous "$S-s$ policy" and is in common use. However, if $I(S)$ has more than one local minimum, the $S-s$ policy is not necessarily optimal, as may be seen by examining Fig. 1.

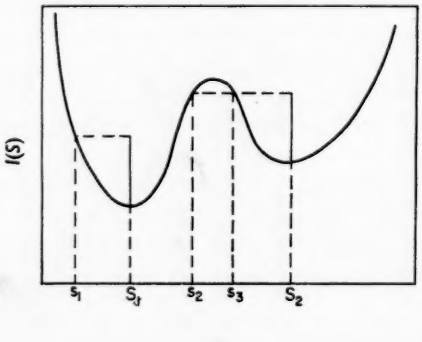

FIG. 1. *Example of non-optimal S, s policy.*

If $I(S)$ has the form in the figure, the following policy minimizes costs over *one* period. [Recall that we are assuming inventory can only be reduced by customer demand, i.e. $S \geqslant s$.]

$$S = S_1 \quad \text{for} \quad s \leqslant s_1, \qquad S = s \quad \text{for} \quad s_1 \leqslant s < s_2,$$

$$S = S_2 \quad \text{for} \quad s_2 \leqslant s \leqslant s_3, \qquad S = s \quad \text{for} \quad s \geqslant s_3.$$

It is not too difficult to see inductively that if $I(S)$ has this form, so do the successive $f_n(s)$ and hence the optimal policy over any period cannot be a simple S–s policy. From a practical viewpoint, the S–s policy may be more universally applicable than the mathematics indicate. If inventory can once be reduced below the level which produces *the* minimum $f(s)$, it will never again rise above this level and we will only be concerned with s_1 and S_1. We have assumed that excessive inventories could not be reduced, except by waiting for demand to take its normal course. There are many practical ways of "artificially" reducing inventories, and we would need to explore them carefully.

Numerical computations. In practice, the probability distribution of demand is likely to be obtained from a frequency count of past experience. We are likely to have the discrete probability $p(x)$ of a demand x (in suitable units). Thus we may as well work with discrete functions when we wish to obtain numerical values.

For costs in the first period we have

$$f_1(s) = I(s) \qquad \text{if we do not order,}$$
$$= I(S)+k \quad \text{if we do order.}$$

If we assume $I(S)$ has only one local minimum, then it occurs at S_1 where

$$\Delta I(S_1-1) \leqslant 0 \leqslant \Delta I(S).$$

Thus if $s < S_1$, we will produce $S_1 - s$ provided that

$$I(s) \geqslant I(S_1)+k,$$

i.e.
$$I(s)-I(S_1) \geqslant k,$$

i.e.
$$\sum_{x=s}^{S_1-1} \Delta I(x) \leqslant 1.$$

Hence the critical value of s, say s_1, above which we will not produce, is given by

$$-\sum_{x=s_1}^{S_1-1} \Delta I(x) \leqslant k \leqslant -\sum_{x=s_1+1}^{S_1-1} \Delta I(x).$$

It follows that to compute s_1 and S_1 we need only know the first difference of $I(x)$. This is fortunate because it is often easier to compute $\Delta I(x)$ than $I(x)$. Once s_1 and S_1 are known, we can compute

$$f_2(s) = \min_{S \geqslant s} \left\{ I(S)+g(S-s)+\sum_0^\infty f_1(S-x)p(x) \right\}$$

by first finding the value S_2 of s which minimizes the right-hand side, assuming $S < s$, and then computing a value s_2 of s, above which no production would be required. The computations may be made with formulas very similar to

those used for $f_1(s)$; we find we need only compute $\Delta f_2(s)$, which turns out to involve only the first difference of $f_1(s)$ and not $f_1(s)$ itself.

A convenient form for computation, for $S > s_2$,

$$\Delta f_2(S) = \Delta I(S) + \sum_{x=s_1}^{S_2} \Delta f_1(x) p(S_2 - x).$$

Having obtained $\Delta f_2(S)$ for $S > s_2$, we can find s_2 by summing the differences in the same manner as we found s_1.

The process can be iterated until s_n and S_n settle down to steady values. As a numerical example, consider a situation where inventory holding costs are proportional to the size of inventory and the time for which it is held. Shortage costs are also proportional to the amount of shortage and the duration. Once a month a decision is to be made of how much to order; if an order is placed, it is filled instantaneously, but there is a fixed cost k associated with placing an order.

A model of this situation, which ignores the set-up cost, is discussed in Chapter 4 of Churchman, Arnoff and Ackoff's book.[3] They show that for $0 \leqslant S < \omega$,

$$I(S) = C_1 \sum_{x=0}^{S} p(x)\left(S - \frac{x}{2}\right) + C_1 \sum_{s+1}^{\omega} p(x)\frac{S^2}{2x} + C_2 \sum_{S+1}^{\omega} p(x)\frac{(x-S)^2}{2x}$$

where C_1 is the cost of holding a unit in inventory for a month, C_2 is the cost of being short a unit for a month, and ω is the largest demand which can occur during a month (i.e. $p(x) = 0$ for $x > \omega$). For this model, when $0 \leqslant S < \omega$,

$$\Delta I(S) = (C_1 + C_2) \sum_{0}^{S} p(x) + (S + 1/2) \sum_{S+1}^{\omega} \frac{p(x)}{x} - C_2.$$

It can easily be shown (although Churchman et al., do not do so) that

$$\text{for } S < 0, \quad \Delta I(S) = -C_2; \qquad \text{for } S \geqslant \omega, \quad \Delta I(S) = C_1.$$

If $C_1 = 1$, $C_2 = 20$, $C = 50$, and $p(x)$ is as shown below,

x	0	1	2	3	4	5	6	7	8	9	10	$7\omega = 0$
$p(x)$	0·05	0·07	0·07	0·09	0·10	0·13	0·18	0·12	0·08	0·06	0·05	0

then we compute values of s_n and S_n as shown in the table.

Although s_n remains fairly constant at 2, S_n has not quite settled down after 11 iterations. However, it appears that a reasonably good approximation to the limit of S_n as n tends to infinity would be about 23. Thus we conclude that the long-term average costs can be minimized by the following ordering rules:

(1) Place an order only when the stock is less than or equal to two.

(2) If an order is placed, order enough to bring the stock up to twenty-three.

In order to appreciate the effects of considering the long-term average costs, it should be noted that the average monthly demand is just over five. Thus there would be a set-up about every four months, and shortages would occur at about the same frequency. If we only consider costs during the month following our decision there would be a set-up about every two months and shortages would occur almost every other month.

TABLE 1. SUCCESSIVE VALUES OF S_n AND s_n

n	S_n	s_n
1	7	−1
2	12	2
3	17	2
4	21	2
5	25	1
6	26	1
7	21	2
8	22	2
9	23	2
10	24	2
11	23	2

A convenient rule of thumb for terminating the computations is to stop when the values of S_n and s_n begin to repeat themselves or when we have passed the first local maximum and minimum. If the latter, a reasonable approximation to the limiting value can be found by averaging the maximum and minimum. Thus we might have stopped at $n = 8$ and taken the limiting values as $1/2\ (21+26) \simeq 23$ and $1/2\ (1+2) \simeq 2$.

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An Integrated Project for the Design and Appraisal of Mechanized Decision-making Control Systems

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I. INTRODUCTION

In the past, the main interest in the control of industrial processes has centred on process control and production control. Process control is the maintenance of certain physical quantities between given bounds or in given relationships to each other. This is achieved by regarding the plant as a mechanism with a given transfer function and adding feedback elements of a suitable kind to achieve the control of the physical quantities involved. It is, in fact, an industrial application of the theory of servo-mechanisms.

Production control is concerned with the construction of a delivery schedule that can be met by using certain rules for passing different batch items through a complex network of batch processes.

In those batch processes which involve a stochastic time-element, another form of control is necessary, the control of congestion. Congestion occurs, of course, in any multistage stochastic batch process, and several mathematical theories have been developed to study simple cases of these. Many industrial situations involve processes in which the process times will be dependent upon the waiting times of batches in the queues prior to these processes. No theory is available for this situation and the purpose of this paper is to describe a project for dealing with it.

In the steel industry, attempts are made to complete the processing of steel in its initial hot condition. However, as the material passes through the various processes, it cools and ultimately, it is necessary to recover the lost heat by passing the steel through heating furnaces. The period in these furnaces will depend upon the length of time the material has taken on preceding stages. This lengthening of the total process time in periods of congestion means that, under certain conditions, it is possible to maintain a more rapid flow of material through the system by delaying the start of certain processes. In more complex plants, in which there is a multiplicity of possible routes for the steel through the plant, it is possible to minimize congestion and maximize the rate of flow by a scheduling procedure. This scheduling, however, is of quite a different nature from that concerned in normal production control problems.

The main significance of this type of interaction between process and waiting times is the positive feedback effect, because as congestion increases, it leads to process times increasing, thereby reducing the effective capacity of the plant,

increasing congestion, ultimately leading to a serious reduction of output. The problems are complex in that this positive feedback takes place with delay, giving rise to historic effects. These historic effects make effective control more difficult.

Congestion control often has several conflicting goals. These centre around the need to minimize waiting times and so reduce process times and the need to maximize the utilization of the process machinery, implying low probabilities of zero queues. A further goal is often to regulate the flow of material to some continuous process occurring as a later stage in the system.

Thus, although an individual decision may lead to an immediate satisfaction of some goal, because of the historic nature of the system, it may later lead the plant to an unsatisfactory state and make future decisions more difficult. Thus, the individual decisions have to be taken as part of a strategy of decisions. This strategy may have as its objective the attainment of long-term goals, quite different from the short-term ones. The long-term goals are often the maximization of average output and the minimizing of the amount of plant used.

This situation has many analogies with the problems studied under the title of Dynamic Programming,[1] but it differs from the situations covered by that technique in that there are no well-defined equations for the outcome of a process as a function of the decisions taken.

The problem is complicated further by there being often many long-term goals just as there were many short-term ones. The balancing of these long-term conflicting goals by objective methods is fraught with difficulties and, in many cases, it is not profitable to attempt to do this. In fact, it may be argued that the function of management in an industrial plant is to take decisions about the balance of these long-term goals. It is for this reason that industrialists are taking an increasing interest in the new science of cybernetics which, through the concept of homeostasis, is giving an adequate method of tackling this thorny problem.

The method of attack outlined in this paper proceeds on the following principle. A control system is designed which attempts to achieve a certain balance of a set of local goals. The balance of these goals is described by a set of parameters and by varying these parameters, differing control systems can be applied to the plant and each will lead to certain levels of attainment of long-term goals. The problem of achieving the "required" balance of these is reduced to determining the "optimum" values of these parameters.

No choice of the parameters in these proposed control systems will give a description of the present-day methods of running the plant. Any initial values chosen for the parameters are unlikely to be satisfactory, and to avoid the waste of production that would arise from experimenting with a real plant to obtain optimum values of the parameters, it is necessary to make a plant model on which these experiments can be conducted.

The appropriate model is a simulation model of the plant operated by a digital computer. This model attempts to give a realistic representation of the

physical behaviour of the plant. To prove the validity of this model, the present control procedure is imposed upon it, and when this gives a correct simulation, the proposed control systems are imposed and the experiments carried out.

Since it is intended to apply this technique to a wide variety of plants, a general computer programme has been developed, which, in terms of a specially designed simulation language, allows the economic building of simulation models of any industrial plant.

Before these simulation models can be completed, data about process times in the plant are required. The collection of these data from ordinary records is unlikely to be reliable and so special data-collecting equipment has been designed which enables this to be stored in the form of punched paper tapes which can be analysed on a computer into the appropriate form required by the simulation programme.

The control systems envisaged will require to have knowledge of the starting and stopping times of processes which must be automatically fed to a control computer. Full details of the equipment required for this aspect of the project are not described in this paper.

The further description of this technique is helped by referring to some examples.

2. TWO TYPICAL EXAMPLES FROM THE STEEL INDUSTRY

The main outline of the total process flow in a steel works is independent of the process used for manufacturing the steel, and is given in Figure 1. After the steel has been made in the appropriate type of furnace, it is transferred to a ladle ("tapped"), which is then used to fill a set of ingot moulds. These moulds are grouped in "casts" and according to the process there may be one or several casts made from each batch or "heat" of steel. After casting, or "teeming", the ingots and their moulds are taken to have the moulds removed or "stripped". After stripping, the ingots are transferred to a reheating furnace or "soaking pit" to recover lost heat and to equalize the temperature of the ingots. The length of time spent in this soaking pit depends on the "track" time from teeming to charging to the pit. After heating in the soaking pits, the ingots are transferred to a primary mill and rolled to an intermediate product, which then passes to a finishing mill which produces the final product. In some cases, a second process of reheating is necessary between the two mills. After the finishing mill, there are various finishing processes such as cutting to length, straightening, etc. Some of these are hot processes, but others are delayed until the steel has cooled on cooling banks.

The major points of congestion are:

(i) In a multiple furnace plant, the crane capacity for handling the ladles often causes delays in tapping the furnaces.

(ii) Cranes in a melting shop have, in addition to the teeming task, a cycle of ancillary activities which often cause delay on teeming a cast.

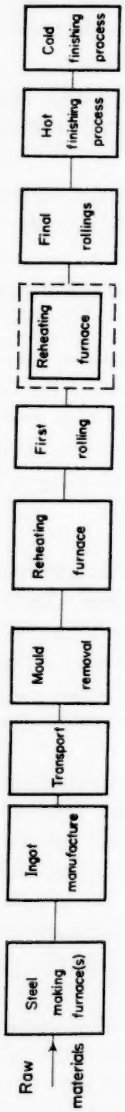
Raw materials
Steel making furnace(s)
Ingot manufacture
Transport
Mould removal
Reheating furnace
First rolling
Reheating furnace
Final rollings
Hot finishing process
Cold finishing process

FIG. 1

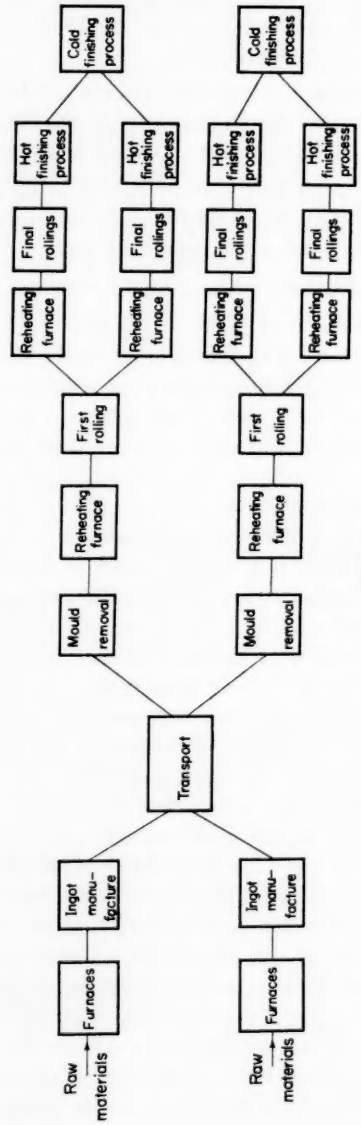
Raw materials
Furnaces
Ingot manu-facture
Raw materials
Furnaces
Ingot manu-facture
Transport
Mould removal
Mould removal
Reheating furnace
Reheating furnace
First rolling
First rolling
Reheating furnace
Reheating furnace
Reheating furnace
Reheating furnace
Final rollings
Final rollings
Final rollings
Final rollings
Hot finishing process
Hot finishing process
Hot finishing process
Hot finishing process
Cold finishing process
Cold finishing process

FIG. 2

(iii) In most steel works, the stripping facilities are limited and delays prior to stripping are common. The lengthening of the track time by this delay and that caused in transport mean that the necessary soaking time is increased. Attempts to cut soaking time lead to a deterioration in the quality of the product.

(iv) The mills are subject to interruption, to change the rolls and for other maintenance, which prevents the removal of material from the soaking pits which then become a source of congestion.

(v) The finishing mill is frequently of lower capacity than the primary mill, and the rate of working of this latter mill has to be irregular to give the required flow to the secondary mill.

(vi) Where secondary reheating furnaces are used, their limited capacity is a further cause of congestion.

(vii) If casts have a very long track time, they would occupy the soaking pit so long that they would cause congestion. To prevent this, such ingots are put into a cold stock and used to charge the pits during starting conditions. However, excess of cold "make" has to be charged during normal running to the pits when it causes worse congestion than it originally prevented.

We may now turn to the two examples.

The first of these is a simple system in which a single Bessemer converter is used as the source of steel. This steel-making process has a short cycle time (10–15 minutes) and uses liquid iron as its raw material which is supplied at irregular intervals from the blast furnaces and kept in a heated reservoir known as a "mixer". One crane is used both to transfer liquid iron to the mixer and from the mixer to the Bessemer and this dual role sometimes delays starting the production of steel in the Bessemer.

The work cycle of the one crane which handles the ladle containing the steel from the Bessemer has a key effect in determining the track time of a cast. Each heat from the Bessemer produces one cast. There is no sale outlet for cold stock and so all that is produced must be reintroduced into the system. There is, however, an outlet for the product of the primary mill. No reheating facilities are available between the two mills and this implies a tighter control on the heating function of the soaking pits. The main form of defect in the product is caused by inadequate heating of the ingots in these pits.

A secondary problem is introduced by the existence of a small arc furnace which utilizes the scrap material produced by the main process and feeds into the system at irregular intervals with an average of about four hours.

The second example concerns a works which is, in effect, two plants combined in one. The outline of this is shown in Figure 2. The sources of steel, in this case, are two multiple open hearth furnace shops fed from unlimited supplies of molten iron and scrap. These two "melting" shops can be used to feed two

mill systems—one a plate mill using large ingots, and the other a section mill using small ingots. In addition to these primary sources of hot steel, there are also preheating furnaces at the mills for bringing cold ingots to an intermediate temperature. There are temporary storages for hot ingots (known as "gjers") which conserve the heat of the ingots better than if these were allowed to stand in the air. There are reheating furnaces between the primary and secondary mills in both systems.

3. AN OUTLINE FOR THE PROPOSED METHODS OF CONTROL FOR THE TWO EXAMPLES

In this section we show for the two examples, the type of control system we envisage, illustrating the parameters of the control system that will be used in the way described in the introduction.

The first example is an illustration of a plant with a single process flow and in which the control procedure merely regulates the start of the production of a new cast of steel in the Bessemer according to the state of the rest of the plant. There are two local goals; these are to minimize the time till the ingots can be removed from the soaking pits and to maintain a steady cold stock. Cold stock is produced as explained earlier or by failing to manufacture steel of the required specification. Such steel must be introduced as cold stock when any product of the actual specification obtained is required.

An experimental relationship between the track time of a cast, the soak time and the probability of a defective product has been established. This enables, at any given level of percentage defect, a necessary soak time to be calculated for a given track time. This relationship is the basis of the control. The level of defective make permitted (which defines the soak time as a function of the track time) is the first parameter of the control system.

If the track time is exceedingly long, the necessary soak time is correspondingly long, and causes an effective reduction in capacity of the soaking pit. It is therefore necessary to specify some maximum track time beyond which a cast will be consigned to cold stock. This is a second parameter of the control system.

It is necessary to start the Bessemer so that the expected time that the steel will be ready to roll is low. The lower this time is made, the greater the possibility of hold-ups to the passage of steel and the greater variability of the "ready time". The variance (mean square error) of the "ready to roll" time is used as a third parameter. Subject to this upper bound on variability, the Bessemer start time is computed to minimize the "ready to roll" time.

This control calculation is complicated by the possibility of the arc furnace discharging casts into the system, but does not affect the principle of the calculation. As events (terminations of processes) take place on the plant, the optimum time is calculated, and the calculation is repeated, adjusting the charge time until the ultimate charge time falls behind clock-time when the charging starts.

This process will produce a certain amount of cold stock and the control system has to arrange for its introduction back into the system. This is done whenever the number of hot casts in the system falls below a fourth parameter value.

The second example illustrates a more complex plant in which route decisions are necessary. The steel manufactured in the melting shops has to have its mill destination settled and this must be done at or before teeming because of the two sizes of ingots used. The bunching of furnace "taps" can be alleviated by adding more liquid iron to a furnace when it is ready to tap and refining for a further period, and this constitutes a second decision required.

The third decision is a charging decision to be made each time a charging crane becomes available. This decision involves deciding if any charging operation should start yet, and if so, from which source (hot ingots from the furnace, preheated ingots, ingots in a gjer, or cold stock), steel shall be taken and to which destination (pits, gjer) it shall be delivered.

The goals in this example consist of minimizing the idle time of the mills due to the shortage of adequately heated steel, maximizing the throughput of the pits so that the number of pits used can be minimized, regulating the rate of accumulation of cold stock as ingots or intermediate material and the maintenance of delivery promises on certain qualities of steel.

The control procedure is based on a schedule calculation which allocates a provisional future history for each cast in the system (including some yet to be charged to the furnace) whenever a decision is required. The decision is taken which implements this schedule. As the plant conditions change, the schedule is revised. The calculation uses as parameters, the number of furnaces active in each shop, the number of soaking pits in use in each mill, certain priority classifications of different qualities of steel, tolerances of soaking time over the minimum necessary, the average proportion of hot steel sent to each mill and the required balance of cold stock at the two mills. It also includes some arbitrary priority rules concerning the charging of steel to the pits which can be altered as another "parameter" of the control.

This is a lengthy calculation and precautions are taken to ensure that it is only revised when there is evidence that significant changes will occur.

After passing the primary mills, the steel is dealt with in ingots and parts of ingots. To eliminate the need for micro-programming of the flow of this steel, the plant beyond the primary mills is regarded as part of the environment of the main plant impressing a demand for steel on the primary mill which is described by statistical laws.

4. AN ABSTRACT MODEL OF THE CONTROL SYSTEMS

This is illustrated in Figure 3. The plant is treated as a Black Box with two groups of inputs. The first represents the state of the market, secular trends and random variables. The second group represents the effect of the control system and these

are given as the output of a control mechanism. One set of inputs to this mechanism are the values of the various parameters.

Both the control systems described above involve a knowledge of the distributions of various process times, and these are additional inputs to the control mechanism. These statistics may vary with time as plant conditions change, and evidence is gathered from the output of the plant to maintain continually up-to-date values of these statistics.

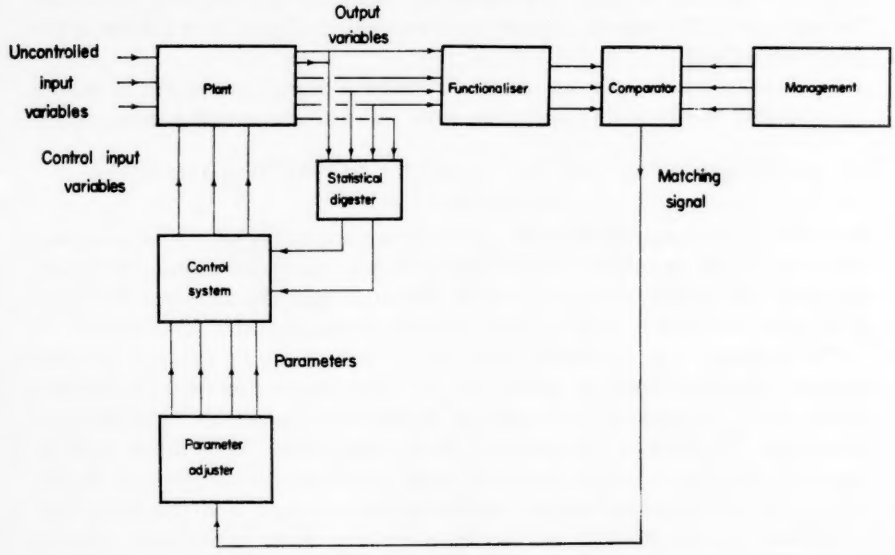

FIG. 3

The instantaneous output of the plant is not of long-term interest and various functionals of this are calculated to present the performance of the plant to management. These functionals usually take the form of simple time averages.

The management is represented as a Black Box whose outputs are the desired states of the goals. These are compared in a comparator with the actual achieved levels of the goals (the functionals of the plant output) and according to the match or mis-match of these, a signal is sent to an input-controlling unit associated with the parameter inputs.

For this complex feedback system, it is necessary that the rates of changing the parameters are adjusted to the rate at which the plant functional outputs themselves respond to changes of these parameters. It will be necessary to conduct experiments to determine exactly the relative rates of response of these two parts of the system.

In general, the outputs of the plant will be continuous functions of the parameter values, and these must be made to change continuously and not to

make rapid random jumps. However, the directions in which to move these parameters are not indicated by the type of mis-match and so a technique is used in which the movements of the parameters in their appropriate phase space are continuous, but suffer random changes of direction.

The whole process has certain similarities with the evolutionary operational technique[2,3] using management as a metricizer of the plant performance.

As explained earlier, the validity of this technique will be established on simulation models, but there is a theoretical interest in the necessary conditions for such a complex adaptive system as illustrated in Figure 3, to behave in the predicted fashion.

To throw some light on this matter, a device is being constructed to enable experimental set-ups of this kind to be made with artificial complex mechanisms.

5. A MACHINE FOR EXPERIMENTING WITH ARTIFICIAL COMPLEX SYSTEMS

The complex systems studied with this machine are highly redundant electrical networks. These have been chosen because they are easily fabricated, the inputs are easily controlled, the outputs easily measured and the networks will have a fast response time so that the time scale can be considerably accelerated.

The networks are fabricated from units, each of which consists of three elements in a star formation, which are used as the connecting elements between the nodes of the network. Networks of $3n$ and $3n+1$ nodes can be made from assemblages of $\frac{1}{2}n(3n-1)$ and $\frac{1}{2}n(3n+1)$ units respectively. Each unit is made of turret construction terminated with an octal valve base and carrying one double triode. The connecting elements are chosen from a range of simple linear and non-linear circuits designed so that the minimum direct impedances between two nodes is high enough to limit the total current drain on the input nodes to a designed level. Each unit is designed so that the net energy input is very low, which ensures that the range of nodal voltages are roughly bounded by the range of input voltages.

A pre-wired arrangement of octal sockets is provided, so that two networks of up to 16 nodes can be assembled. This arrangement allows the 32 nodes to be split up into a larger number of smaller independent networks if required.

Any of the nodes of the network can be selected by patch cords as inputs and any of the remaining nodes can be used as outputs.

The inputs available are:

(i) from steady sources which are manually adjusted with potentiometers to simulate secular trends,

(ii) a set of rapidly fluctuating inputs to simulate the random behaviour, and

(iii) inputs which simulate the searching process associated with the parameter values in the scheme of Figure 3.

These latter inputs derive from the voltages on capacitors which are either charging or discharging. The direction of movement is settled by the states of

flip-flops. When a change of direction in the parameter searching phase space is required, the flip-flops are set to a new random configuration. A random digit generator has been constructed which enables random bits to be produced at a rate of 5,000/sec.

Various packaged units are being developed to create functionals of outputs and these are compared with one another in a further packaged unit, which gives as output the absolute value of the difference of its inputs. This can be fed to a differentiator and the resulting signal passed to a discriminator (comparator with zero). An "or" gate arrangement then can detect if any response recedes from its goal.

A cathode ray tube monitor is incorporated to study the movements of any selected variables.

6. THE GENERAL SIMULATION PROGRAMME

This part of the project has already been reported,[4] and will be treated more fully elsewhere.[5] Here only a brief outline of the principle of working will be given.

The plant is regarded as a set of machines, each with a set of states. Any change(s) of the state(s) of (a) machine(s) is (are) regarded as an event and the simulation moves from event to event. At any moment of time, machines are grouped together in activities, which endure for a sampled time, and then become free, after a possible change of state, to regroup with other machines in further activities.

A language has been developed for naming the machines systematically and describing their states and the times of changes in these. Tests on the states and changes to them can be made in statements in the language.

The tests of states necessary for an activity to start are written down in this language followed by the actions necessary to represent the activity starting and to commit the machines to the new states for the duration of the activity. Further statements are written down describing any changes of state which occur at the conclusion of the activity.

This is repeated for each possible activity and these are assembled together with the initial conditions of the plant by a translation programme which prepares the input for the simulation programme.

Facilities are provided in the language for instructions to print out any necessary information and to arrange for alternative courses of action when machines break down.

The language enables lists of items in queues and other passive elements to be introduced quite simply. There are facilities to extend the possible statements by the introduction of specially written sub-routines which are given code-names in the language and can use the states of machines as parameters.

The language has been designed and the programme written so that:

(i) simulations written in this language will operate as fast as any special simulations,

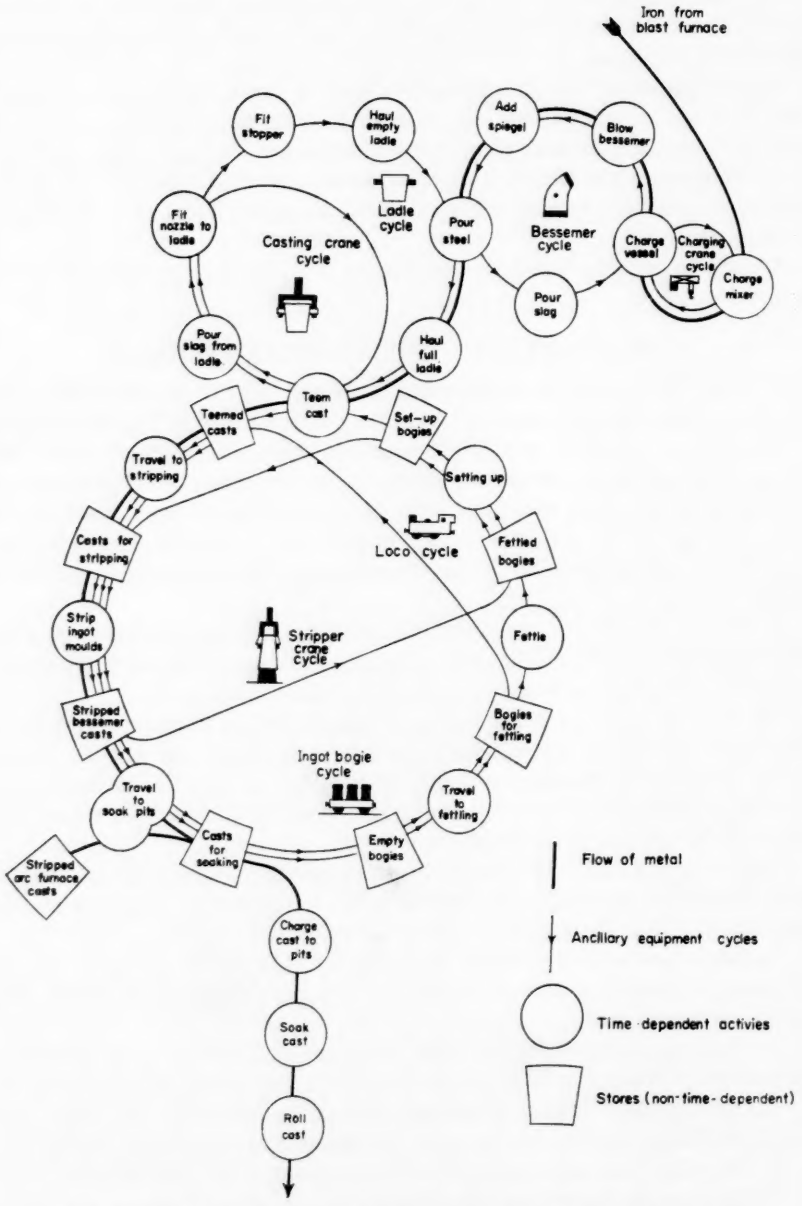

FIG. 4. *Simplified flow diagram of activities. Acid Bessemer steel-making plant.*

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TABLE 1. LIST OF MACHINES

	P	Q	R S T	U	V	W
1		Store Content	Iron Supply			
2			Mixer		Content	
3			Charging Crane			
4	Making Code		Bessemer Vessel	Operation		Making Batch
5			Ladle 1	Operation		
6			Ladle 2	Operation		Ladle in use
7	Teeming Code		Casting Crane	Operation		
8			Locomotive	Position		
9			Stripper Crane	Operation		
10	Code		Arc Furnace Bogie	Position	Track 1	Track 2
11	Code		Bessemer Bogie 1	Position	Track 1	Track 2
12	,,		,, ,, 2	,,	,,	,,
13	,,		,, ,, 3	,,	,,	,,
14	,,		,, ,, 4	,,	,,	,,
15	,,		,, ,, 5	,,	,,	,,
16	,,		,, ,, 6	,,	,,	,,
17	,,		,, ,, 7	,,	,,	,,
18	,,		,, ,, 8	,,	,,	,,
19	,,		,, ,, 9	,,	,,	,,
20	Code		Soaking Pit 1	Operation	Track 1	Track 2
21	,,		,, ,, 2	,,	,,	,,
22	,,		,, ,, 3	,,	,,	,,
23	,,		,, ,, 4	,,	,,	,,
24	,,		,, ,, 5	,,	,,	,,
25	,,		,, ,, 6	,,	,,	,,
26	,,		,, ,, 7	,,	,,	,,
27	,,		,, ,, 8	,,	,,	,,
28	,,		,, ,, 9	,,	,,	,,
29	,,		,, ,, 10	,,	,,	,,
30	,,		,, ,, 11	,,	,,	,,
31	,,		,, ,, 12	,,	,,	,,
32		Track Time	Pit Charger	Operation		
33			Arc Furnace		Counter	Cold Stock Counter
34	Rolling Code		Rolling Mill	Operation	Over-soak	Rolling Batch
35			Fettler			
36			Arc Furnace Queue	Position		
37		Counter	Meal Timer			
38			Production Controller			

CELLS NOT ANNOTATED are used as working spaces.
Rows 50–79 are used for non-time dependent machines
Rows 100–149 are used for parameters
Rows 150–179 are used for schedule details.

(ii) the process of codification reveals errors or omissions in the specification which must cover the behaviour of the plant in all possible conditions,

(iii) the possible statements in the language are indefinitely extendable.

The first step in writing a simulation is to prepare a flow diagram showing all the possible activities in the plant. A simplified version of this is given for the Bessemer example in Figure 4.

From this diagram a list of machines is prepared as in Table 1. The various cells of the diagram are then translated into the language. A simple example is given in Table 2 which has been annotated in ordinary English to illustrate the conciseness of the language used.

TABLE 2. TYPICAL ACTIVITY STATEMENT

Activity Number C4
Activity Name Pouring Steel from Bessemer Vessel

Subject	θ	Object	
U4	=	2, A	If the Bessemer vessel has finished operation number 2 and is now available,
U(n+5)	=	n/2 5, A	search over the two ladles to find one available which has finished operation number 5.
W6 RAN P152	→	n	If a ladle is found, record which one and calculate an activity time for activity C4.
U(n+5)	→	O, D, Then B5	Change the operation number of the ladle and dispatch it for the calculated activity time ensuring that activity B5* is engaged when the ladle has finished operation number 0.
U4	→	3, D	Change the operation number of the Bessemer vessel and dispatch it for the calculated activity time.

* Activity B5 is hauling the full ladle to teeming.

The adequate simulation of any plant requires the specification of activity time distributions and other random variables which must be collected from plant records. The description of the simulation reveals these requirements. The data is then collected using, if necessary, the semi-automatic data recorder described in the next section.

7. SEMI-AUTOMATIC PLANT DATA RECORDER

This machine has been described elsewhere[7] and a brief description is given below. To match the simulation language, the input is in the form of messages concerning the state of machines in the plant and the time of changes in these states.

The machine consists of a series of independent control stations of varying form which translate the input data into a suitable coded form. These coded

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data, together with the time of occurrence, are then punched on to paper tape under the control of a main unit (Figure 5).

Each control station consists of a number of coding devices, a marking device and a scanning mechanism which is under the control of the main unit.

The coding devices in the simple case of direct manual control consist of a series of switches each of which has 10 positions. They may be rotary switches or multi-push button switches. Each switch assembly represents a machine, and the ten positions of the switch represent ten possible states of that machine, only one of which may occur at any time. There may be up to 21 machine switches in one control station.

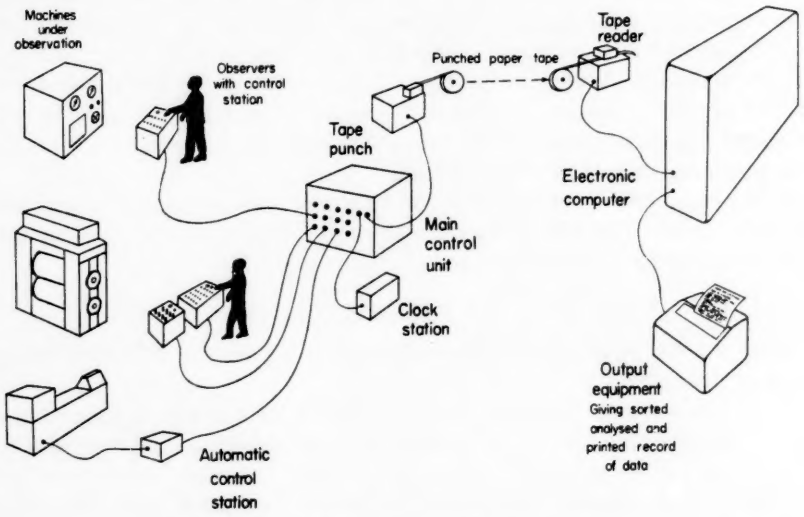

FIG. 5. *General scheme of use of data recorder.*

The main unit contains a synchronous electric clock which gives an impulse every $\frac{1}{8}$ of a minute, the basic unit of time measurement. After the required code has been set on any control station, the marking device is operated. Immediately after the next clock impulse, any marked control stations are scanned by the main unit and the code configurations contained therein are punched on to tape. Every punching sequence is headed by the "clock-time" of marking and the name of the control station.

There are two types of manual control station. A Type 1 station has high priority access to the punch, being recorded, when marked, as soon as possible. A Type 2 station has access to the punch only when no Type 1 stations are marked and is used for the insertion of parametric information.

It is possible to replace the code switches in a control station with a set of relays which can be operated by signals fed direct from the plant under observation.

In such an automatic control station, the marking is achieved by a relay which senses a change occurring in any of the code relays.

For cases where some information is required from a point remote from the remainder of the plant, a special impulse operated control station has been developed which permits of the passing of information on up to 20 states over a pair of telephone wires. This is capable of operating over distances of up to two miles.

Where it is required to record short time intervals with a much greater accuracy than $\frac{1}{8}$ minute, another special control station has been provided. This contains a form of stop clock recording up to 6 minutes in $\frac{1}{10}$ second intervals which is started and stopped by the equipment under study. Each time it is stopped, the interval measured is punched out on tape as the code message.

The punched tape produced is designed specifically for computer analysis and the codes used have been chosen to ensure, as far as possible, the detection of any recording errors. A special computer programme provides for the checking and initial sorting of the data and this can be followed by any desired standard programmes for data analysis.

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A five-hole code is used and this can be reproduced over any standard teleprinter channel. This allows the transmission of data automatically from outlying factories to a central computer.

8. CONCLUSION

The project will be developed in 9 phases and use a variety of differently trained staff in these stages:

(i) *The study of the plant processes.* This is carried out by engineers in collaboration with staff trained in simulation model building who can then proceed to (ii).

(ii) *The development of a simulation model of the plant.* This process has been described earlier.

(iii) *The development of a decision control procedure.* The decision control procedures are, in a sense, an arbitrary feature of this technique and, as far as possible, are made rational extensions of the present empirical rules used by plant management, and are designed in collaboration with them.

(iv) *The collection and analysis of plant records.* The collection of data is handled by Work Study teams and the necessary analysis is specified by the simulators and extracted from the paper tape records by programmers on the digital computer staff.

(v) *The translation of simulated models and control procedures to computer programme.* This is achieved by co-operation between the simulators and the programming staff. The initial proving requires the imposition of the present-day control procedures which are not necessarily rational or objective. It is, therefore, not possible to simulate these with a

computer programme and the simulation is adapted to print out the necessary information when each decision is needed, and this is presented to members of the plant management who make the necessary decision. Thus the simulated model is run by the plant management as if it were a real plant. This serves the dual purpose of proving the simulation model to the project team and inducing faith in the soundness of this technique among the plant management. After this stage, the simulation reverts to free running with the proposed control procedures.

(vi) *Evaluating goals.* This involves discussions with the plant management to obtain an accurate set of long-term goals.

(vii) *The design of the experimental goal searching programme.* The final random searching of the parameter phase space is intended to make the control adaptive and is not a very efficient procedure for determining optimum conditions from an arbitrary start. The Box evolutionary technique which makes a systematic search of the phase space in the neighbourhood is more efficient and statisticians in the group adopt Box's experimental designs to this purpose.

(viii) *The design of equipment for real plant control.* This stage has not yet been reached in any applications of this technique, but will involve co-operation between the laboratory staff and the project team, the plant management and equipment manufacturers.

In anticipation of the design of this equipment, experiments in the design and fabrication of "on time" digital control are being conducted by the laboratory staff.

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Operational Research and Management

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(*Translated by* A. W. Swan)*

The purpose of this paper is to discuss and if possible to dispel the industrialist's misunderstanding of operational research. The author lists some of the varied ideas held by industrialists of operational research including several which are contradictory, as for example (a) That operational research is new and revolutionary; and (b) That operational research proves nothing new and has always been practised without being so named.

A number of suggestions are then made and developed, which should lead to a better understanding. These include, clearer definitions of operational research and a better adaptation of the specialized literature to the needs of industrial management, with particular emphasis on the publication of real rather than synthetic case histories. Headings of sections are:

1. The attitude of industrialists.
2. The exact nature of operational research.
3. Problems, criteria, models.
4. The practice of operational research in business.

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1. THE ATTITUDE OF INDUSTRIALISTS

THE idea of operational research held by a number of industrialists and engineers is still far from being clear, and this is the cause of quite a series of special reactions varying from an instinctive mistrust to active obstructionism.

Among the attitudes which result from insufficient information, the list below gives some which seem characteristic, although admittedly the list is not complete.

The Exact Nature of Operational Research

(a) Operational research consists in turning management into mathematical formulae.

(b) Operational research consists in resolving management problems by electronic computers or similar machines.

(c) Operational research consists of an assembly of mathematical techniques.

The Practical Range and Utility of Operational Research

(a) Operational research is a revolutionary innovation in the science of management or on the contrary, operational research proves nothing fundamentally new, and has always been practised without being so named.

(b) Operational research is given up to theoretical developments which have no practical application. Actual practice is very different.

(c) There is no need to undertake complicated studies. All problems can be best resolved by common sense and good judgment.

* Based on a paper published in the Journal of the Federation of Belgian Industries, August, 1959.

The Conditions of Application of Operational Research

(a) Operational research is expensive; it is a luxury that only large firms can offer.

(b) The role of management is constantly to take decisions as speedily as possible. Management is therefore unable to use operational research to advantage because operational research tends to be slow in reaching conclusions.

(c) The general lack of statistical data and relevant cost figures and the doubtfulness of forecasts are such that an operational research study cannot be built other than on a foundation of sand, and its conclusions will therefore always be useless.

Several of these notions reflect part of the truth, while, at the same time, distorting it, notably the last two given above. They are, however, all incorrect, and arise from a profound misunderstanding of the real nature of operational research. Rather than refute them one by one, we propose to try in the following paragraphs to put forward in a positive manner the fundamental characteristics and practices of operational research, hoping that at least we are not adding to the confusion on the subject.

There are many reasons for this widespread lack of understanding of the nature, the possibilities and the limits of operational research. Three of these are:

(a) The absence of a single clear and universally accepted definition which points exactly to the content of operational research.

(b) The lack of adaptation of the specialized literature to the needs of industrial management.

(c) The confusion, even among many practitioners, between operational research and certain mathematical techniques which form part of the specialist's kit of tools.

Specialist literature; books, journals, unpublished papers, etc., can be divided into two classes. Those relating to the scientific developments of the techniques that can be used in operational research. Such developments are absolutely essential for the progress of this discipline and constitute the base of its progress. These publications are, however, by their very nature, directed to the practitioners in operational research or to the engineers and economists who wish to study the mechanics of operational research. They are not directed at industrial managements and are not likely to enlighten them.

Articles which one might call "missionary". These are intended for the general information of higher ranks of management and are almost always built round an imagined management problem. Many of these examples are so naïve, so simplified, so far from the technical and economic reality of the life of a business enterprise, that they may well irritate and annoy the persons most ready to receive the principles they put forward.

There is to-day an enormous gap in the available literature on operational research. There are remarkably few actual "case histories" which stress the following aspects:

(a) The general context of the problem.
(b) The precise definition of the problem, with notes on the difficulties encountered in making this definition; later and final definitions of the problem.
(c) Choice of criteria of efficiency to take into consideration, and the reasons for the choice.
(d) Factors taken into consideration in the study, and the reasons for the choice of these factors.
(e) Concepts adopted in regard to costs and profits, with reasons for their adoption.
(f) Statistical, economic and accounting data available; examination of their validity. Correction of data by observation or by experiment; measures taken either in the absence of certain data or, in the case of uncertainty, of the validity of the data supplied.
(g) The choice of the model or models adopted, and why this choice was made; the methods of work followed.
(h) Difficulties encountered in the course of the study; obstacles, dead-ends, withdrawals, modifications to the model.
(k) Conditions of validity of the solution; studies of the degree of this validity.
(l) Application of the solution; results obtained.
(m) The practical aspects of the method of carrying out the study; its cost and duration.

Such examples do exist in the published literature, but they are rare and always in a very condensed form.

A great many operational research problems, very varied in nature, have been solved successfully, and this is happening every day in every type of business. Unfortunately, however, the documentation of most of these case histories remains inaccessible because it is considered to be confidential, which is, after all, not so very surprising. It would be desirable, however, in the general interest, to persuade industrial companies to emerge from this too complete reserve and develop a pool of information relating to real case histories for management. Even if curtailed and transformed to some extent by summarizing and simplification, the real case history can easily be distinguished from the artificial problem which has been worked out to illustrate particular theories.

2. THE EXACT NATURE OF OPERATIONAL RESEARCH

Operational research can be defined by the convergence of: (a) a research region; that of the preparation of management decisions at various levels and in various contexts; (b) The application of a method of approach which is rigorously scientific, a method which uses quite an arsenal of techniques and

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often employs speedy and powerful methods of calculation. Operational research, therefore, is above all dependent on an attitude of mind in regard to the problems put by management. This is a label which has led to the paradox that a distinguished scientist, the French physicist-economist, ALLAIS, has denied operational research any separate existence, while praising its content and employing it to carry out important practical applications.

One of the most important characteristics of operational research is its essentially practical aim; it is not directed to the academic study of the optimum conditions which would lead to the evolution of any given organization, but is concerned with the preparation of concrete decisions, i.e. it prepares for action. This action, once started, will have consequences which in their turn will modify the conditions under which the organization works, and there is thus an effect of feed-back which is characteristic of all operational research followed by action, and which has therefore an affinity with certain aspects of Cybernetics.

In our opinion the definition sometimes given of operational research as being the science of the preparation of decisions is too wide. Many decisions can be and are taken on a simple basis of a nature which may be ethical, technical or otherwise. Operational research must have some material for investigation, and this applies to operational problems that are affected by many factors, some of which may be random and have complicated interactions, so that the optimum solution is far from being obvious, or detectable by plain common sense or by empirical reasoning. This description applies to most problems of operation and policy which face the management of every type of organization, be it large or small, and also to other levels of organization in human society.

It is not therefore a question of converting the management of a company into equations nor of trusting to electronic computers in order to use technico–economic problems as an amusing exercise in mathematical virtuosity. Operational research is rather concerned with all the techniques and other tools which are likely to prove useful and to project the spirit and philosophy of scientific research into a domain which until very recently, with rare exceptions, lacked this kind of thinking.

What is fundamentally new in all this?

It is not difficult to cite typical applications of operational research which were worked out many years ago without bearing the name of operational research; such, for instance, were the studies made in Copenhagen fifty years ago by Erlang, who was an engineer, on the saturation of the telephone network of the Danish capital. Nevertheless, certain characteristics of what one calls operational research are genuinely new, as follows:

Applications. Operational research has a point of view which in the formulation of policies and in preparing for complex decisions introduces a spirit of systematic and rational research, and in many cases gives results which are far better than those reached through empirical methods. The correctness of this conclusion has been demonstrated experimentally since the experience

gained during the last war. The extension of this method takes into systematic consideration the scientific notions of probability, of risk, and of uncertainty in the management of business.

Zones of permissible error in the management of business are necessary to keep the cost within bounds. This reduction of management tolerances is the result of many factors of world-wide origin which are continually multiplying and which represent the loss of economic flexibility resulting from continued technological progress and mechanization and the consequent degrees of freedom left available to directors.

Methodology. There has been developed fairly recently quite a series of analytical techniques, which are particularly well adapted to management problems. Several of these techniques were, in fact, suggested by the problems.

It is now realized that it is possible to transpose scientific methods from their original field to a different field which apparently has no connection with the original, and how efficient such a transposition often is.

Powerful and speedy machines have been developed which make it possible to-day to make calculations and experiments in a relatively short time and at a reasonable cost which could have not been tried a few years ago.

3. PROBLEMS—CRITERIA—MODELS

Problems. The problems capable of being solved by operational research are extremely numerous in the most varied fields of human endeavour and at all levels. India, for instance, has created within the past few years an Operational Research team at Government level in order to help the economic development of the country.

At such a level, operational research shows strong affinity with Econometrics, many of the techniques being common to both disciplines. A great difference, however, and one which is, from our point of view, essential, is that Econometrics is concerned with a disinterested scientific research; it studies quantitatively the relations between economic factors with the job of establishing functional models which will explain their mutual interactions. Operational research, on the other hand, looks for practical answers to precise problems, and has the job of recommending action which will react on the set of conditions originally found and eventually modify them.

It is obvious that there is no shortage of complex management problems in businesses of all sorts and sizes, and operational research should be directed to these problems and not on whether or not an investigation is going to be interesting or whether it is going to use a particular technique.

The only valid question in connection with any given management is the following: "What are the management problems which are affected by a complex series of factors whose solutions are, at present, left to intuition or to the application of traditional methods?" Problems of this nature are particularly important from the economic point of view.

If the answer to all the questions arising from the management of a business can be easily supplied without uncertainty or doubt, so much the better; there is no point in undertaking vain investigations. On the other hand, it is almost too obvious to state once again that the rational study of a problem is never a luxury but always a necessity, so that one can provide for the maximum efficiency in the resulting actions.

The perils of common sense. It is reasonable at this stage to ask if common sense combined with experience is not, in the long run, the most adequate instrument for trying any kind of policy or management problem.

Experience and common sense will certainly be counted as among the most important qualifications for a manager of a business, and operational research does not pretend to be a substitute either for experience or for common sense, but to use them to the best advantage.

It is necessary all the same to beware not of common sense but of the assertion of any given individual that he has lots of common sense. Descartes once said, "Common sense is certainly the most widely spread gift in the world, for I have never heard anyone complain that he did not have sufficient common sense".

Too often, so-called "common sense", brandished as a banner, is, in fact, a screen for intellectual laziness, for slackness, or for prejudice. This kind of "common sense" can do a great deal of harm. Human judgment does not, bearing in mind how the brain functions, proceed intuitively to a correct synthesis of a problem if the factors involved are numerous and random; it has to depend on analysis, and on a series of logical steps in reasoning.

Empirical methods and intuition can often give brilliant results, but it is imprudent to rely on them too much or too often. The methodical application of analysis, of logic, and of quantitative study—in other words, of the scientific approach, cannot but considerably improve management decisions; it is the experiment ascertaining this truth of evidence which is at the base of the scientific development of operational research in the world. After all, why should problems be studied less when they become larger and more complex?

Strategy and tactics. It is possible to distinguish in the life of any business the management decisions which relate to everyday work of a repetitive character and the exceptional decisions which relate to the general future of the company. These two types of decision are sometimes called tactical and strategical. In the first category are, for example, problems of purchase, of production planning, of stock control, and of inspection; in the second are problems that arise when one is considering whether to make a company larger or smaller, modifications to a distribution network, investments, sales, or financial policy.

These two types of problem naturally call for a different type of treatment, above all in the presentation of the proposals for action. Tactical problems imply the establishment of rules for optimum decisions, of interpretation and of application which will be sufficiently simple even for relatively subordinate staff; rules to which changes can be easily made as the factors change.

A study of strategical problems will be translated into a series of recommendations which will explore not only the zone of optimum solutions but the zones of sub-optimum solutions. In all these cases, an operational study must by its very object be carried out in such a manner that the conclusions are available in the time required before a decision for action has to be taken; if it is not ready in time its utility is zero. The width and precision of the study must then be adjusted in terms of the amount of time available.

At this stage there arises the problem of urgent decisions, and it is obvious that really urgent decisions cannot be reached through a lengthy study so that it is sometimes necessary to resort to intuition and experience. All the same, many problems are not as urgent as they are made to appear; a spirit of insufficient provision for the future and excessive centralization of the power of taking action are the two factors most responsible for this state of affairs. Many people are so bound in by the routine of their daily work that a problem only holds their attention when it is projected on the screen of this routine, by which time it is too late to act effectively and their decisions are hasty and the solutions ineffective. What would not be gained in business in time, in trouble, and in money by a movement towards the future of one's bundle of preoccupations? However this may be, the shorter the time allowed before a decision must be given, the vaguer will inevitably be the conclusions of any preparatory study. It is nevertheless preferable to obtain in the time specified, answers which are relatively inexact but which are still an approximation to the correct answer than either to wait too long for the precise results required or to remain in total ignorance.

Criteria. The precise definition of an operational research problem and of its limitations is often difficult, delicate, and subject to adjustment. This stage is one of the most important sections of a study and it is impossible to give it too much attention.

To this definition of the problem is linked the choice of the criterion of efficiency which conditions it. An optimum has to be found which is a function of a criterion that is clear and free from ambiguity. It is seldom possible to express an optimum for several different criteria. Even in limiting the field of discussion to private companies working in a free market and looking for a profit, the choice of criteria is often far from obvious.

We give below a series of criteria which can be considered for the conditions described above.

1. Maximization of the absolute profit over a certain period.
2. Maximization of the relation of the absolute profit to the capital invested.
3. Minimization of a total cost over a certain period.
4. Guarantee, with a high probability of realization of an absolute profit at least equal to a certain given value.
5. Minimization of a risk.
6. High probability of obtaining in a fixed time a given proportion of a certain market.

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7. Guarantee, with a high probability of the survival of the company.
8. Elimination of a competitor.

This list is far from being complete. Other criteria can be established on such bases as the need to have liquid funds, the prestige of the company, the stability of employment, etc.

In many problems the correct criterion is obvious. In other cases there can be discussion and the choice of a criterion can itself become the object of operational research. Even in limiting oneself to the notions of profit and of cost, one runs into difficulties relating to the definition of these terms. The notions of profit and of marginal cost are of prime importance, but it is still essential to define them precisely in the context of a given problem, as they are never so simply or so easily handled as one might think.

To sum up, not only must the choice of the criterion applicable to a decision be the best possible, but the concepts of cost, of profit and of value must be chosen correctly or there may be a very serious overall miscalculation.

The above is just as important for a quick and superficial study as for a full-scale operational research investigation. A series of adjustments, not always easy, may be necessary to the available basis of information, and it is extremely dangerous to omit such adjustments.

Models. Operational research is characterized by the establishment of models which represent the operations being studied. Every investigation calls for the establishment of a model or more frequently of a series of models. The model is intended to represent in one form or another, the mutually complex relations which exist among all the factors, all the variables, which have to be taken into consideration in the investigation. As the objects for operational research are essentially practical, the models used must be realistic. They must take into account all the really important factors with their important characteristics.

In the most general case, it is rare for a problem of operational research however small it may be, to be represented by a single model; it will rather be represented by a chain of models interlinked in such a way that a main assembly chain is supplied by sub-assembly chains which themselves result from a convergence of subsidiary chains, and so on. Since each problem calls for the design of an appropriate model, it is absolutely essential not to use prefabricated models. Here we sometimes see opposition between "theory" and "practice". Actually, there is no fundamental difference between the two. Theory is, after all, condensed practice, synthesized and generalized, but there is good and bad theory as there is good and bad practice.

In businesses, even those which are the most complex and most subject to uncertainty, events that behave in an apparently chaotic or arbitrary manner usually show a certain deterministic pattern of a statistical character and this pattern can be discovered.

An operational research model, has, in general, the form of a network of quantitative or sometimes qualitative functional relationships among different

variables, and therefore has often a mathematical appearance. Many of these relationships are of a probabilistic nature. The mathematical notation is particularly useful for the compact expression of such relationships and for their manipulation in such a way as to extract from them all the logical conclusions desired. Operational research is not a branch of mathematics, but uses the mathematical and statistical tools that are particularly appropriate to its needs. Here again, to oppose the employment of mathematical language to "practice" is an absurdity and comes from a total misunderstanding of the nature of mathematics. Mathematics provides a logical method of transformation, based on an appropriate symbolization which creates nothing by itself but which deduces logical conclusions from certain premises. All quantitative reasoning automatically makes use of models even if these are of an elementary character, but it is not always possible to express a phenomenon in a simple form.

The mathematical manipulation of the model in a search for optima often involves massive calculations, leading eventually to the use of an electronic computer. Certain models will be studied analytically, others by speeded-up experiment or simulation, and this may sometimes call for the use of analogue computers.

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Information available. Analysis of sensitivity. The manipulation of a model and search for the optimum imply the preliminary determination of quite a series of numerical values attached to certain parameters and to certain coefficients; and these may be technical, commercial, economic, or financial. The validity of these numerical values is vital, for upon it depends entirely the results of the investigation. For this reason the greatest possible care must be taken in their assembly, in their critical analysis, and in the pertinence of the concept of cost and profit taken into consideration. Quite a high proportion of the total time required for an investigation is often devoted to this apparently unrewarding but essential activity.

Nevertheless, it often happens, however much care is taken in this task, that the documentation available will be less than that required in quantity or inferior in quality, or both, and furthermore, a large coefficient of indeterminacy may affect certain basic data, such for instance, as forecasts of sales. What is to be done in such circumstances? Since to base the investigation on incomplete and uncertain data is to adventure into the completely unknown, must one abandon the problem? Yes, as postulated, but the problem is still there and it must have a solution whatever the failings in the basic data. Is intuition better than logic and to a methodical approach in such a situation of insufficient information? There is no reason to accept such an assumption.

There is a way of solving the difficulty; it consists of utilizing for each doubtful or uncertain parameter not a single numerical value but a zone of numerical values: for example, instead of defining the annual forecast as a single figure such as 12,000 tons, it can be defined by a range of 8,000 to 15,000 tons.

The calculations resulting from the manipulation of the model must be carried out successively on the basis of different values of doubtful parameters chosen from the ranges selected.

There can be two results from the foregoing procedure; either the optimum solutions are relatively little affected, qualitatively speaking, by the variations in the values of the uncertain parameters, in which the lack of precision in the available information does not bring about too serious consequences and does not result in uncertainty in the solutions. Alternatively, the optimum solutions are affected, qualitatively speaking, by the variations in the values of the doubtful parameters. Under these conditions it is impossible to obtain a single optimum solution and one must be content with an optimum zone.

If the above is true, it underlines the extreme importance of obtaining the most precise and accurate information for a business. Let us suppose that two different values h_1 and h_2 of an important but doubtful parameter h produce different optimum solutions which have an economic value that are respectively equal, in terms of profit for instance, to V_1 and V_2. Let us presume also, that the real value of this parameter is h_1 while the general view is that it is h_2, and that solution 2 is adopted in consequence. Instead of obtaining V_1 or V_2, a result R_2 smaller than V_2 and V_1 will be obtained. The difference $(V_1 - V_2)$ expresses the value of the information corresponding to the gain of precision in the knowledge of h.

Such considerations can be decisive for a business, and thus influence its complete system of information, of statistics, and of forecasts.

Note that it is very dangerous to represent by a single average value the parameters whose values are distributed statistically; these parameters must be represented by the complete characteristics of their specific distributions.

The method which we have just described can be called an analysis of sensitivity. An analysis of sensitivity consists, therefore, in the examination of the variations of the qualitative conclusions of an operational research investigation as a function of the modifications to the values of certain parameters by certain limiting conditions and by certain hypotheses. Although little reference is made to this type of analysis in the specialist literature, and there is curiously little written on the subject, an analysis of sensitivity should be made in every investigation. The conclusions resulting from these analyses of sensitivity are in our view among the most important results of an operational research investigation; they show the reaction of the prosperity or otherwise of the business to the manipulation of certain key factors.

4. THE PRACTICE OF OPERATIONAL RESEARCH IN BUSINESS

Operational research is an attitude of mind in regard to management problems. It does not consist solely of a formal study of a certain number of problems, but must be shown by an intensification of the spirit and practice of the scientific method of reasoning at all levels in management. Operational research must not be the preserve of a small group of specialists.

To repeat, it is essential that operational research in a business should deal with concrete problems and not arise out of the desire to apply a given technique or a given method of calculation.

Investigations which are specifically operational research can only be undertaken by analysts who understand the techniques and the possibilities. This implies, as things are at present, the necessity for the acquisition of a considerable amount of supplementary knowledge by the engineers, economists and mathematicians who become interested in this discipline.

All the same, a mastery of the techniques is far from being sufficient. A good practical knowledge of the economics of the business, and some experience also in the solution of industrial, commercial and financial problems are valuable, as is also the habit of working as a member of a team.

Given the type of problems that are adequate and the fields in which operational research works, it would be extremely dangerous and far from realistic to give an operational research investigation to a young engineer or young mathematician who has just graduated from a university; such a young man has his place in a team, but only as a junior member of that team.

In all cases, and at all stages of development in an investigation, close contact must be maintained between the operational research analyst and, at all levels, the managers concerned with the problem. This is particularly true in the first stages of the formulation of the problem and the choice of criteria, and at the end of the investigation. The problem of communication and of the justification of results is very important and usually far from easy to solve, all the more because in the final presentation of the conclusions the mathematical developments must be left to the end of the report.

In all cases, the responsibility of decision for action cannot be delegated and it remains entirely on the shoulders of the responsible executive. Let us remember that the conclusions of an investigation must always be accompanied by an analysis of sensitivity. It will be essential to translate these conclusions into practical rules of action that are clear and easily applicable to the actual work situation. The savings in time or money, or both, resulting from the application of operational research are generally quite considerable, which is not surprising considering the variety of possible application. Operational research is sometimes, as mentioned earlier, considered as expensive, and is for that reason sometimes not used, but it would be quite illogical to dissociate its cost from its returns. Considered as an investment, operational research usually offers a very high rate of return.

Some remarks made by the American industrialist, T. WARE, in J.O.R.S.A., January, 1959, under the heading of "An Executive's viewpoint" are appropriate: ". . . They have tended to ignore the fact that the salary of one Vice-President for one year could finance a good-sized operational research study, and they do not often ask whether the output of that Vice-President is likely to be worth more to the corporation than an operational research study".

Letters to the Editor

DR. EILON'S paper "Economic Batch-size Determination . . ." (O.R.Q., **10** (4), 217, December 1959) extends the usual treatment aimed at:

(a) minimum cost per piece,

to two further criteria,

(b) maximum profit per batch,

(c) maximum rate of return (on the investment made in producing the batch).

In his discussion he points out that (b) gives large stocks and low turnover, (c) involves higher production costs, and concludes that ". . . the question of a criterion selection . . . has to be carefully and seriously analysed . . .". He seems unaware that the theory is a lot simpler than he suggests, and that the problems which the paper finds so difficult are quite artificial and due entirely to his having introduced two pointless and misleading criteria.

One hesitates to use your space to rehash the known and well-established principles of the subject, but it may nevertheless be desirable to clear up any possible confusion.

First, the object of this exercise is to discover how to make the greatest possible profit. Not maximum profit per batch, nor maximum rate of profit per £1 invested, nor per square foot, nor per British Thermal Unit; just maximum profit, full stop, though the latter may in individual cases imply one or more of the former. In Eilon's case, which is the usual one, the rate of sales is taken as fixed, at so many pieces per year; hence one wants maximum profit per piece. If the sales price is also fixed, this implies minimum cost per piece, i.e. criterion (a).

Eilon's difficulty of taking into account the ". . . relation between input (investment) and output (profit); (and) the time factor (which determines the rate of turnover)", disappears provided one allows for the cost of money before calculating the profit. Surely it is a known and well-established practice that the cost of holding stock in the formula must include the cost of the investment? If money is free, it costs nothing; if it has to be borrowed, its cost is given by the rate of interest and the time over which it is invested; if (as is most usual) money is available but could be profitably used elsewhere, its cost is given by the profit it would have earned had it not been invested in producing the batch.

It follows that in all these cases criterion (a), if properly used by allowing for the cost of money, will give the right answer.

It is indeed true that there are businessmen such as those mentioned by Eilon who ". . . often talk about maximizing the profit for the whole operation . . . (i.e.) profit for the whole batch", and no doubt there are some who think they ought to have maximum rate of return. But it is surely the function of the O.R. worker to tell them gently that they could make some more money if they aimed at a more relevant criterion. In this connection the following questions may be found useful.

(1) (For those who want maximum profit per batch.)
Would you rather make £1,000 p.a. with quarterly batches or £300 p.a. with annual ones?

(2) (For those who want maximum rate of return.)
Would you rather make £1,000 p.a. on an investment of £500, or would you prefer to have a free gift of 6d.? (The last shows an infinite rate of return!)

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ALTHOUGH the paper was not specifically written for the purpose of discussing the merits of various criteria in batch-size determination, I am grateful to Mr. Kerner for his comments, which illustrate only too well the need for meticulous definitions in any optimization process.

Mr. Kerner sets out to clear up any possible confusion by rejecting outright several criteria, each including the word profit in one form or another, and he proceeds to state flatly what he is after: ". . . just maximum profit, full stop". How admirably concise and lamentably vague! This is precisely the sort of statement that begs the question: What *is* profit? It is only too reminiscent of some managerial policy statements: "I want the best possible results". Surely, the whole point of optimization is that we have a clearly defined yardstick for measuring effectiveness, a criterion which leaves no room for ambiguity.

Mr. Kerner eventually concludes that maximum profit per unit should be the ultimate objective. Obviously, if this is his interpretation of "just maximum profit, full stop", then his criterion is well defined and I fully agree that in this case minimum costs per unit should be aimed at. However, this criterion does not tell us anything about the amount of financial effort required to produce a batch, nor about the rate at which investment and profit are realized. I agree that the cost of investment and the loss of other potential opportunities can theoretically be taken into account in the formula for minimum cost batch-size (in which case the term minimum costs per unit becomes a misnomer, really), but I suggest that the criterion of rate of return is a more direct and more realistic method of definition.

Finally, I do agree that the function of O.R. workers is to be concerned with criteria and their outcome. Some O.R. workers prefer the attitude of: "You define the objective, I shall (try to) find the optimal solution"; but it would seem that the study of objectives, values and utilities cannot be divorced from the optimization process. We must appreciate the compatibility of different criteria and their mutual relationships if we are to make intelligent analyses and recommendations, particularly in multi-objective systems, in which reduction of the problems to a one-dimensional measure of effectiveness is not as simple as sometimes suggested. In the case of optimal batch-sizes, it has been demonstrated that some objectives are not synonymous, and it is by no means obvious that the conventional measure of minimum costs is always superior to any other objective. As the profit margin per unit increases, it may well be far wiser to produce smaller quantities than the minimum-cost batch-sizes and thereby achieve a quick capital turnover, which could then be utilized on additional activities.

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A Practical Application of Linear Programming in the Mining Industry

IN the September 1959 issue of the *Operational Research Quarterly* a short note was published giving details of an investigation we have carried out into the transportation of coking coals. It has been pointed out to us that we omitted reference to an interesting parallel to our own work. This is a paper by Mrs. A. M. LAND (Ref. "An Application of Linear Programming to the Transport of Coking Coal", *J. R. Statist. Soc.* A **120**, Part 3 (1957)). Although we used neither the methods of solution nor the formulation of the problem presented by Mrs. LAND, we would like to draw the attention of the readers of the *Quarterly* to this particular paper.

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Abstracts and Reviews

A Comprehensive Bibliography on Operations Research: 1957.

Prepared by the Operations Research Group, CASE INSTITUTE OF TECHNOLOGY, CLEVELAND, OHIO. 123 *pp.* 82.

Reviewing a supplement to a bibliography is rather like reviewing a new edition of the London Telephone Directory—all the best material has been used up.

The first Bibliography of Operations Research prepared by the same group was published by John Wiley in 1958. It consisted of 3,000 entries up to 1956, and included a preliminary supplement for 1957. The present supplement gives more complete coverage for 1957. My impression is that the compilers have made reference to more journals than they did last time, and I was unable to think of anything worthwhile that they had missed.

The method of classification is such that it is more suitable for punched-card than book reference. Entries are in alphabetical order of authors with a ten-digit code to indicate the content. Hence with the book version, unless one can recall authors' names, one must scan every code number in the book; for example, for entries about linear programming one must pick out those code numbers having 03 as the 6th and 7th digits. A long job. Unlike the earlier issue, there is no subject breakdown; again, unlike the earlier issue, there is no mention of the availability of the punched-card form.

The final frustration for someone in this country comes when he tries to get hold of some of the journals referred to!

JOHN BANBURY

Mathematical Techniques in Industrial Engineering. Part I—Linear Programming.

K. J. SHONE.

Institute of Industrial Engineers (India). 70 *pp.*

Linear programming has now been accepted as one of the fundamental techniques of operational research, and in the past two years we have seen an enormous increase in the number of books published on this subject. Most of these books presuppose that the reader, be he economist, mathematician or operational research worker, has a fairly sophisticated outlook and that he is alive to the possibilities of linear programming methods. We should therefore welcome the appearance of any really elementary textbook which may be studied by workers who are interested in linear programming but who, for lack of time or knowledge, find most of the available books unsuitable. The

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book under review, which is based on a series of lectures given by Professor Shone in India in 1958, is of a size and scope to fill this need, but unfortunately it fails to do so.

Before analysing the reasons for this, a brief outline of the contents is as follows. The first chapter deals with the assignment problem, and the next two with the transport problem (twenty-seven pages in all). Chapters 4 and 5 are on the ratio analysis method (twenty-five pages), and the final chapter of thirteen pages is on the simplex method. Whilst admiring the diagram used to illustrate graphical ratio analysis in three dimensions, it is regrettable that a method whose range of practical application is limited to problems with at most three effective constraints should occupy such a large part of the book. This criticism is the more serious in view of the inadequate provision made for further reading.

Indeed, it is from this lack of balance that one can deduce the main cause for the failure of the book. In the original lectures, Professor Shone would doubtless have placed a proper emphasis on the various points as they arose. But during the transition from rostrum to print, this emphasis has become misplaced, and the notes do not in themselves constitute an ideal basis for a book. In addition, they have been insufficiently edited and there is almost no documentation. Although the lack of suitable sources is offered as an explanation for citing only four references, this is exactly the sort of difficulty which must be overcome by the author of any *introductory* text. The book is not free of technical errors and loose terminology (maximum profit is equated to minimum cost without giving any reason as to why this should be so), but although these are annoying, they are of secondary importance compared with the unbalanced structure of the whole.

ALISON DOIG

Operations Research. An Annotated Bibliography. (2nd Edition.)

JAMES H. BATCHELOR.

St. Louis University Press, St. Louis, Missouri, U.S.A., 1959. ix+865 *pp.*

In reviewing a volume of abstracts (as, in fact, this is, since the annotations are a fair indication of the contents and scope of the various publications listed) it is important to assess the collection for what it sets out to be and not to raise objections that it is not something else. The Preface discloses the method of compilation and is, to that extent, disarming, as the book's virtues and limitations are thus indicated at the outset. It contains then, everything appearing in the eleven principal periodicals in the field and in books, papers and collections from other sources and also nearly all the titles given as references in these primary sources, to a total of 4,195 entries in all—a formidable task of compilation for which all credit is due. Much border-line material is necessarily included, and it is likely that most that has been generally overlooked in compiling the

primary sources is missing here. How much this may amount to it is impossible to say but one is reminded how often in other sciences some vital contribution has been neglected for a long period from some accidental cause, like Avogadro's Hypothesis and Mendel's experiments. There is, of course, no other comparable compilation, but the above may at least suggest that the search for previous material should not be limited to this volume; the compiler no doubt never intended that it should be, but to the reader the temptation to accept its virtual completeness will be great.

Individual items in the abstracts are very good of their kind, necessarily brief and, if not without implied expression of opinion, at any rate very close to that ideal; when a random selection of the original papers is consulted the annotations are found to be very good within their self-imposed limits.

With so much that is good it is a pity to have to report that the value of the book is spoilt by the inadequacy of its indexing arrangements. The abstracts are arranged through the book in alphabetical order of first authors and they are numbered consecutively; unless, therefore, the reader knows the names of all authors of papers on the subject he is concerned with, this arrangement is of little help. The index does little to redress the balance as there has been a marked tendency to list titles as they stand; thus there are nearly two pages under "application" ("de" and then "of"); two pages of "on" are followed by five of "operational(s) research".

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These blemishes undoubtedly spoil the present volume; fortunately they can be readily put right in future editions which it may sincerely be hoped will follow regularly.

J. G. COCKBURN

Méthodes et Modèles de la Recherches Opérationnelle.

A. KAUFMANN.

Dunod, Paris, 1959.

This is a striking book and a valuable contribution to the literature on operational research, in distinction from that dealing with the individual techniques used in operational research.

Monsieur Kaufmann had a definite purpose in mind, and he has written a book carrying out his purpose. It is intended for the engineer or other technologist who has had a sound grounding in mathematics and who wishes to learn about the methods of operational research and how they may be applied, and it certainly will be useful to this class of reader. It is not intended for the uninstructed industrialist and is therefore not of a "missionary" character. The mathematics, except perhaps for the final section which is definitely devoted to mathematical research, are not beyond the capabilities of the engineer, physicist, etc., who has included mathematics in his technical training.

The book starts with an introduction which sets out the purpose of the book, and it is worth while perhaps to quote briefly from this introduction.

"A good (O.R.) analyst must be at the same time a very well-equipped mathematician, a statistician, a well-trained economist and a man of experience. Should one advise all others not to become interested in O.R.? We do not think so, for the procedures of this discipline will only be developed as the knowledge of their possibilities becomes known among higher managements. This is not a book dealing with statistical methods or probability theory and it does not include sequential problems, games theory, dynamic programming and the theory of information."

After an interesting chapter that the author calls "Généralités", the first main section consists of four long chapters, devoted respectively to linear programming, queueing, stock control and maintenance and replacement. The second main section follows, and in this section the mathematics of the same techniques (i.e. linear programming, etc.) are developed from a purely theoretical point of view.

In the chapter "Généralités" the definition is given that operational research is concerned with the control of artificial phenomena rather than of natural phenomena, and that it is in fact the "science of the preparation of decisions". This latter definition corresponds closely to that which has been used for some years by Monsieur Kaufmann's fellow-countryman, Dr. Réné de Chantal, who is best known for his connection with the development of work study in France.

From the above definition Monsieur Kaufmann proceeds to a section in which the principle is enunciated that operational research always seeks a mathematical model for the situation being examined, this varying from a simple algebraic equation to a complex set of equations. Two examples are given to illustrate the basic idea of the mathematical model, one being a pendulum, the other an electrical circuit containing an inductance and a condenser. In both cases the fundamentals of these examples are reduced to simple differential equations.

The introductory chapter continues with a note on the important industrial fact that the theoretical optimum for any given solution is seldom attainable, and that the goal which should be sought is the best possible within the practical limits of the situation. The term which seems to have become generally accepted among what Monsieur Kaufmann calls the Anglo-Saxons is "sub-optimum", a term which somehow has a defeatist connotation.

The chapter concludes with a useful division of problems into four main classes, (a) where the analyst is faced by a deterministic universe, (b) where the analyst is faced by a universe where there are random elements and these can be examined statistically, (c) where the analyst is faced by an intelligent competitor (and uses the theory of games), (d) where the analyst is faced by an unknown universe, scientific method is of little avail, and intuition may well be used in its place. It is interesting to note that Monsieur Kaufmann has included (d) in this list.

The first of the chapters dealing with specific techniques is devoted to linear programming, and like those that follow in this section, is simple and clear and contains no mathematics beyond differential equations. The explanations given are of a straightforward character, similar to those that can be found in existing textbooks, but probably the most useful point to the student is that several practical examples are then given, each worked out in detail. These have to do with a programme for a rolling mill, the controlling of a production plan from sales information, the choice of sources of energy in the production of electricity, a dual problem in the sale of food products, transport problems and assignment problems.

The following chapters on queueing, stock control and maintenance are equally practical. Each begins with a clear statement of the theory as it stands at present and follows on with a number of practical examples, each worked out in complete detail, in some cases with a family of curves or nomograph. (Note: "Abaque" is used to signify any graphical device used to assist in making calculations.) A word of caution should be uttered to the English reader at this point; French mathematicians have evolved some interesting expressions of their own which cannot be literally translated into English. "Espérance mathématique" is translated "expected mean value".

The second main section of the book is of equal length to the first and is concerned, as has been said, with the mathematical development of linear programming, queueing theory, etc., already discussed in the first section of the book. This section is quite definitely for the advanced practitioner in operational research who is already familiar with the more elementary notions, and it should be of considerable interest to him as it gathers into some 250 pages a great deal of material, part of which the author willingly states is not original, but which has not hitherto been assembled for convenient reference.

A number of useful tables are given, including the binomial, Poisson, normal and χ^2 distributions, Carson–Laplace transforms, etc.

There is also a helpful bibliography which not only includes books but a large number of papers published in a variety of journals, mostly American, English, French and Italian.

Monsieur Kaufmann disarmingly concludes as follows:

"Some will consider it inopportune or premature to expose methods that are still in process of evolution to engineers and to those who directly or indirectly are concerned with management, but on that basis one would never learn anything for there is always change. One of the most attractive aspects of O.R., as of all research, is the adventure that one finds, in that it has numerous ambushes, but also provides the satisfaction that the solution of a fragmentary aspect of a small problem sometimes brings. But the big problems of to-day are doubtless the small problems of tomorrow."

A. W. SWAN

Simulation: Management's Laboratory.

A student report prepared at Harvard University Graduate School of Business Administration by Simulation Associates, P.O. Box 55, Bradford, Mass., April, 1959. 118 *pp.* 90*s.* (\$12.50 *post paid.*)

This 118-page report has been written for middle and upper management. Its object is to present clearly and simply the ways in which simulation can assist businessmen in their work. Important concepts of the subject are explained; a large number of practical applications of simulation are discussed; and the considered opinions of a wide range of people experienced in simulation are presented.

Its secondary objective is to assist men who are already employing simulation by expanding their ideas about its present and potential applications. The report also discusses problems associated with simulation and the presentation of results to management. The forty-odd cases described include several that have not been previously published; the remainder are drawn largely from published material. The bibliography, which is believed to be the most complete yet published on this subject, is intended for people who are now using or contemplating the use of simulation. Throughout the text, parenthetical numbered references correspond to the numerical listing of books, articles, and papers in the bibliography.

Sections II to VI are concerned with the use of simulation for problem-solving and for business planning. Section II describes the basic ideas of simulation, and explains the need for models. Section III is a detailed discussion of four simulation projects of a complex nature. In each case, the simulation technique and the organizational problems encountered in completing the project have been described. Sections IV and V describe cases in which simulation has been used by industry. (Section IV contains cases in the general area of operating problems, while Section V covers those involving long-range planning and investment decisions.) Section VI discusses management's role in deciding how and when to use the method, how to train men to do the work, and how to evaluate the results.

The next sections of the report deal with simulation as a vivid means of training executives and operators. Section VII describes and evaluates the various management games devised in the last few years. Section VIII treats a real-time simulation used to train operators.

In Section IX, three important areas where simulation is being used as a research tool are described. These areas are economics, with particular reference to the improvement of models of the economy; the study of the company and its environment as a unified system; and the study of human behaviour viewed as systems.

The concluding section deals with current work being done to advance the usefulness of simulation in business. Three appendices are also included.

Appendix A is a brief note on computers, with reference to their use for simulation. Appendix B is a glossary of terms commonly found in the literature on simulation, and is designed to assist readers who may wish to study some of the references in more detail. Appendix C is a selected annotated bibliography, which lists 204 books, articles, and papers on the subject of simulation. It is divided into: General Theory; Industrial Applications; Military Applications; and Related Reading. The articles range from the very simple to the very technical.

[*Copies of this report can be obtained from B. O. B. Williams, c/o John Williams of Cardiff Ltd., East Moors Road, Cardiff. If cheques are sent to B. O. B. Williams they should be made out to him.*]

G. M. HESELTON

Resource Allocation on an East Anglian Dairy Farm. (Occasional Paper No. 6.)
C. S. BARNARD and V. E. SMITH.

Farm Economics Branch, School of Agriculture, Cambridge University, April, 1959.

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This paper is a study in the application of linear programming on a small, 84-acre dairy farm. The object is to show how a problem concerned with maximizing the net revenue in the short run may be formulated and a model constructed, and to examine further the hypothesis that there is considerable scope for economic expansion on farms by a better allocation of resources among already existing opportunities.

The example is worked out in detail and the Optimum Plan shows a 25 per cent increase in profit over the Present Plan. This increase is shown to result from the sum effect of a number of changes taken together and not from any one change. The modality of the farm studied supports the thesis that farmers are not, by any means, allocating their resources ideally.

Queueing in the System G2/M/1.

P. D. FINCH.

Acta Math. Hung., **10** (12), 241–247 (1959). (In English.)

Balking is defined as the departure from the queue of a customer who finds n customers ahead of him. Barrer has used the term "impatient customers" in a previous paper (*Operat. Res. Quart.*, **5**, 644 and 650 (1957)) dealing with a related problem. The problem treated here is the single-server queue with a general input and exponential service on first-come-first-served basis; the maximum size of the waiting facility can have any value, and the balking process can be random or deterministic. Finch proves that if the input distribution is not a lattice distribution then a limiting distribution of the balking probability exists and is independent of the initial state. He obtains the state probabilities for the balking system.

The results of Haight (*Biometrika*, **44**, 360 (1957)) for Poisson input and infinite waiting capacity and Takács (*Annal. Univ. Sci. Budapest, Sectio Math.*, **1**, 73 (1958)) for deterministic balking are shown to be special cares of the general formulation of this paper. Takács, it must be noted, has also treated this case for the many-server problem. Barrer's problem is not listed, but since it deals with balking as a function of the time spent in waiting, it too should be a special case of this general problem.

Remarks on the Dam with Poisson Type Inputs.

J. GANI and N. U. PRABHU.

Aust. J. Appl. Sci., **10** (2), 113–122 (1959).

The paper consists, basically, of two parts: (1) a review of the continuous dam model with simple Poisson inputs, capacity K, a steady release except when the dam is empty and overflows recurring whenever the dam content exceeds K; and (2) a brief analysis of the dam with continuous inputs which are infinitely divisible distributions of the Poisson type. Both problems can be interpreted as queueing problems and formulated by using the Kolmogorov equation for the transition distribution function. The first problem is a special case of Takác's (*Acta Math. Hung.*, **6**, 101 (1955)) integro-differential equation for analogous queueing problems associated with telephony. The Kolmogorov equation reduces to

$$\frac{\partial}{\partial t}F(z,t)-\frac{\partial}{\partial z}F(z,t)=-\lambda[F(z,t)-F(z-h,t)],$$

where $F(z,t)$ is a cumulative distribution function

$$F(z,t)=Pr\{Z(\tau-t)\leqslant z\,|\,Z(\tau)=z_0\}.$$

$Z(t)$ is the content of the dam, at time t, h is the incremental input and λ is the output rate. The solution of the infinite dam problem leads to a more direct stationary solution of the finite dam problem ($K<\infty$) which is equivalent to the solutions given by Moran (*Aust. J. Appl. Sci.*, **6**, 117 (1955)) and Gani (*Biometrika*, **42**, 179 (1955)). The second treatment results in the Kolmogorov equation

$$\frac{\partial F}{\partial t}-\frac{\partial F}{\partial z}=-\int_0^\infty \lambda(u)\{F(z,t)-F(z-u,t)\}du,\quad o\leqslant z\leqslant K.$$

A stationary solution for the infinite dam exists when the mean input per unit time is less than the mean outflow rate per unit time. Detailed results are promised at a later date. Although the mathematical treatment is thorough and almost complete, there is no indication of the practicality of the formulations in so far as actual water storage problems are concerned. For this the cited work by Moran and other works by the same author are required. (See: Moran, P. A. P., *Aust. J. Appl. Sci.*, **5**, 116 (1954) and *Quart. J. Math.*, **7**, 130 (1956).)

Optimum Cloth Width and Machine Size in Textile Manufacture.

D. BRUNNSCHWEILER.

J. Text. Inst., **50**, 574 (1959).

In textile manufacture there is some latitude in the choice of the width of cloths made for some purposes, and in the size of machines that will be used. With these widths and sizes are associated different costs of manufacture, and optima can be calculated. These problems are discussed with reference to beam warping, multiple-needle tufting, warp-knitting, and weaving.

Optimum Discount Rates for Cloths of Sub-standard Width.

M. SANDELIUS and J. LINDBERG.

J. Text. Inst., **50**, 583 (1959).

Pieces of cloth made to a specified width vary in actual width. If the garment manufacturer adopts a simple cutting routine involving the use of "lay-markers", pieces that are narrower than the "lay-width" involve the increased costs associated with hand-laying, pieces that are wider involve the costs associated with the excess of material that is wasted. For a given distribution of cloth widths there is an optimum lay-width which minimizes the average cost per garment. The impact of this situation can be transmitted to the cloth manufacturer by charging a discount on pieces that are narrower than a standard width. For a given discount, standard width, and width distribution, there is an optimum average width—in excess of the standard width—that minimizes the total cost to the manufacturer per piece. Further, if the garment and cloth manufacturers both work to their respective optima, there is an optimum discount rate that minimizes the total cost per garment. These optima are determined on the assumption that the width distribution is normal.

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Linear Programming.

E. R. PALA, J. TOLRA and J. DE LA CRUZ.

Dyna, **34**, 160–181 (*March* 1959).

A brief review of the functions of linear programming is followed by a theoretical treatment of some of its techniques, said to be sufficiently detailed to be applicable to actual cases met in industry. The theoretical treatment is applied to problems in connection with an automatic lathe plant, at the works of Hispano Olivetti S.A., in which 47 groups of machines were available to carry out 670 tasks. Optimum and empirical solutions are worked out and compared.

Premises for Operational Research in Italy.

S. RICOSSA.

Dyna, **34**, 160–181 (March 1959).

A general outline of the principles of operational research is given, with some reference to its application in Italy. Operational research is considered as a

rational means of determining problems of organization, as a method of computation for obtaining numerical solutions to quantitative problems. The article concludes with a worked example of linear programming taken from *Journal of Industrial Engineering*, 1957, Nov./Dec.

The Use of Computers for Economic Planning in the Petroleum Chemical Industry.

G. S. GALER.

Computer J. (October, 1959).

The complexity of the petroleum chemical industry leads to difficulty in the formulation of economic marketing and manufacturing plans. Shell Chemical Company Limited has gained considerable experience in the use of linear programming for this purpose, and now regularly uses the Ferranti Mark I* in Amsterdam to produce its medium-term programmes. The Simplex algorithm readily allows small modifications to be made to initial conditions without the need for extensive recalculation; this fact is being fully exploited in order to speed the provision of up-to-date information on optimum operating conditions. There are plans for a considerable expansion of this kind of work as the new Ferranti "Mercury" computer becomes available in London during 1959.

Strategy in Research.

D. S. MCARTHUR and J. J. HEIGEL.

Esso Research and Engineering Co., Abingdon, 1959.

This paper discusses strategy in empirical research. Several research teams have done empirical research on electrical analogues of research problems where a random, normally distributed error was introduced to measurements of the dependent variable. The success attained with different strategies has been observed. Designed experiments are "better" than the one-variable-at-a-time strategy. However, the "best" strategy is not known.

The work pointed up mistakes we are apt to make in doing research:

We aren't always bold enough;
We are apt to get in a rut;
We are too easily fooled by the error;
We don't always eliminate the unimportant variables first; and
We don't always know when to quit.

Some of the "elements" which will probably be found in the "best" research strategy are suggested.

News and Notes

The Scottish Council (Development & Industry), Future Resources Enquiry and Action, Scotland

THE Scottish Council (Development & Industry) has launched an enquiry to consider primary resources—as an inter-related group—people, land, water fuel and energy, minerals, fish and wildlife, and forestry. The project will be steered by a "Committee on Natural Resources in Scotland", whose remit is to:

(1) form a fact-finding study group on Natural Resources (including human resources), and make recommendations;

(2) run a series of operations based on the findings of the study group, which might lead to the provision of a permanent body in Scotland, such as a centre for resource development transactions and authoritative information.

It is one of the objectives of the Committee to see established an organization, which can provide basic resource information and be of use to the research and development departments of industrial companies.

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News of Members

Changes of Appointment

R. Chittleburgh—Previously with Dunlop Rubber Co. Ltd., Birmingham, is now with Dunlop Rubber Co. (India) Ltd., West Bengal.

Dr. J. O. Cutress—Previously with the National Coal Board, Newcastle, is now with the Central Electricity Generating Board, Portishead.

G. A. Hutcheson—Previously with I.C.I. Ltd., is now with the National Union of Manufacturers Advisory Service Ltd., London.

E. Kay—Previously with NAAFI, is now with Standard Telephones & Cables Ltd., London.

R. A. Leaver—Previously with the British Transport Commission is now with A. E. Reed & Co. Ltd., Maidstone.

P. A. Longton—Previously with BEA, Ruislip, is now with Shell-Mex & B.P. Ltd., London.

A. J. Mayne—Previously with A. C. Nielsen & Co. Ltd., Oxford, is now with the Electronic Computing Laboratory, University of Leeds.

B. H. P. Rivett—Previously with the National Coal Board, London, is now with Arthur Andersen & Co., London.

Dr. M. G. Simpson—Previously with the National Coal Board, London, is now with Messrs. Touche, Ross, Bailey & Smart, London.

J. K. Wyatt—Previously with Glacier Metal Co., Wembley, is now with British Petroleum Co. Ltd., London.

New Full Members of the Society

J. W. Grant—International Computers and Tabulators Ltd., London.

D. J. Thruston—International Computers and Tabulators Ltd., London.

Report on the Symposium on Mathematical Programming, Santa Monica, California, March, 1959

IN a lecture to the 1959 Harrogate Conference of the Society, Dr. S. Vajda of the Admiralty Research Laboratory described some of the topics discussed at the Santa Monica Symposium, which he attended. The following notes have been prepared from Dr. Vajda's material.

Most of the work of the symposium was concerned with linear programming and associated techniques. The five main areas of discussion are described briefly below.

1. *Recent novel applications of linear programming.* Two new types of problem in which classical L.P. methods could be used were discussed.

The first of these (presented by Oliver Gross) dealt with a problem similar to the well-known allocation problem. This is to allocate resources in such a way that the minimum value of the individual allocations is maximized. This is analogous to determining the maximum strength of a chain, and can be solved very simply.

The second concerned bid evaluation, in which bids are made offering a variety of items, the bids each being subject to various constraints such as the total value of the accepted bid, the quantities of each item taken, etc. This has presented much more formidable analysis, and is not yet solved.

2. *New computational techniques in linear programming.* With the growing need for simplification of the computational procedures in order to handle large-scale problems, several new techniques, all based on the simplex procedure, were reviewed.

Most of these are only applicable under special conditions, but the Gross Section method, so called by J. J. Stone, who developed it, is applicable to all L.P. problems. In this method, the constraints are introduced sequentially, and a series of interim solutions is obtained and used to obtain the next until the final solution, conforming to all the constraints, is obtained.

Another method, developed by the Socony Mobil Oil Company, was used for a joint production and distribution problem. This is the problem of minimizing the total costs of production at several plants (where the volume of production can be varied at each) and the subsequent costs of distributing products from plants to customers.

3. *Theoretical developments in linear programming.* Several recent advances were discussed, among them being some work on the determination of the maximum number of basic solutions in an L.P. problem by W. Jacobs and E. Schell, and Professor Tucker's work relating the duality of L.P. problems to geometrical orthogonality in multi-dimensional space.

4. *Discrete solutions in linear programming.* Advances in the theory of handling discrete L.P. problems have been made, particularly at the London School of

Economics, and at Princeton University, and Ralph Gomory of Princeton demonstrated his method of solution.

The application of discrete L.P. methods to the travelling salesmen problem in which more than one salesman is involved was discussed, and Harvey Wagner demonstrated a method of tackling machine scheduling problems in which the sequence of operations is important.

5. *Non-linear programming*. Three principal categories of handling problems in which the constraints are non-linear were covered at the symposium.

The first covered iterative methods of solution to such problems by replacing the non-linear constraints by tangential hyperplanes. These include the methods due to J. E. Kelley and to E. W. Cheney and A. A. Goldstein.

Secondly, Ascent methods, similar to those developed by G. Zoutendijk and J. Rosen were reviewed. In these, preferred routes of moving from points on the boundary surface to more nearly optimum points are identified and the solution again obtained step-wise.

Finally, Philip Wolfe and others presented work using Lagrangean methods of solution, based on a theorem by Kuhn and Tucker. These methods involve finding saddle points and then adapting the simplex programme in the light of these results.

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Open Days at BISRA

AT the recent Open Days at the Battersea Laboratories of the British Iron and Steel Research Association (BISRA) interesting displays represented the work of the Operational Research Department. This department is headed by Mr. R. H. COLLCUTT and is divided into three Sections as follows:

(i) Human Factors (6 graduates led by Dr. S. LANER)

(ii) Computer Applications (10 graduates led by Mr. R. J. TAYLOR)

(iii) Operational Investigations (13 graduates led by Mr. R. R. P. JACKSON).

The Human Factors Section is concerned with problems where the human element is a major factor and work in the field of accident prevention and ergonomics was exhibited. A description was given of the application of statistical method to the interpretation of works accident records. Most charts of accident rates give no indication of the significance to be attached to the sometimes violent fluctuations in rate, and comparisons between departments on the basis of such charts tend to be misleading. If the numbers of accidents are plotted on charts similar to the quality control chart the stability of each department's accident rate can be checked and different departments can be compared directly. Another interesting study concerning the use of safety boots and shoes was illustrated. It appears that the facilities for easy payment and the actual wearing quality of the boots are more important factors conducive to their use than their effectiveness as a protective device. Mention was made of an experiment being carried out in a number of works to test the effectiveness of

displaying posters intended to induce the worker to modify his method of carrying out his work. It appears from these experiments that certain well-designed posters advocating a particular procedure (in this case a procedure tending to be more safe than alternatives) may have a positive and measurable effect for at least a period of 3 months.

Other studies of this Section which were described included measurements of the effect of heat on the individual and the effectiveness of protection against heat. Laboratory apparatus used in experiments in connection with the design of control points was also exhibited. The Section has given valuable advice to the industry on the design of crane cabs, seats and controls.

The Computer Applications Section, besides providing computing services to the department and the Association on their Pegasus digital computer, are undertaking studies of the applications of digital computers in the steel industry so as to advise management how to decide where and when to use them. A description was given of the specification of an on-line information collection and display system for a steelworks, the subsequent design of which was undertaken by the Association's Electrical Section. Another project was concerned with the use of a digital computer in the production allocation office of a major steelworks. The customers' orders are committed to punched cards or tape and all subsequent processing to produce production schedules, costs, invoices and advice notes, to answer customers' enquiries, and to inform them of delivery dates, etc., is undertaken by a computer. This system shows considerable economic advantage over current manual or punched card techniques. Mention was also made of more ambitious schemes to take over by computer the major part of all the clerical routines at a steelworks associated with production scheduling and control, cost and financial accounting, sales, stock control and accounting, maintenance records, and the production of statistics for management.

Another project described was the on-line use of a digital computer in conjunction with the saw, where the lengths of as-rolled hot steel section are cut to the lengths required by customers. Although men are capable of mentally selecting very efficiently the lengths to cut (wastage was only about 1·5 per cent of output) it has been shown that the use of a computer would provide substantial financial advantage, even when two computers were employed to cover breakdowns.

Demonstrations of the Pegasus computer included the use of the "Montecode" interpretative scheme designed by the Section. The "Montecode" is a computer programme in autocode form developed for use in preparing programmes for doing Monte Carlo runs on the computer.

The Operational Investigations Section undertakes all operational research studies which are not primarily personnel or computer problems; a major part of their work is confidential to individual member companies.

Descriptions were given of many different projects including the following: a case where raw material stocks were reduced by over 50 per cent; the

programming of the repairs of open hearth furnaces to develop methods of planning applicable to any works in the country; studies by simulation of the organization of a melting shop; the use of linear programming in choosing between various steelmaking processes for a new steelworks; decisions regarding re-equipment in various types of works; the choice between rolling to customer's ordered length or rolling standard lengths for stock with subsequent cutting to the customer's requirement. The display was summarized in a panel which stated in general terms where operational research would help management in their decision-making, and this proved most helpful.

London School of Economics

OPEN DAY OF THE RESEARCH TECHNIQUES DIVISION WEDNESDAY, 16 DECEMBER, 1959

THE Director of L.S.E., Sir SYDNEY CAINE, welcomed those attending the meeting and Professor KENDALL gave a summary of the direction in which research had been done during the ten years for which the Division had existed.

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"Research on Dynamic Economics"—Professor A. W. Phillips

PROFESSOR PHILLIPS described his work against the background of traditional economic theory in a competitive commodity market where there was no control on production, consumption or price. The competitively determined price was held to establish equilibrium conditions, acting as an automatic correcting or equilibrating device. One example of such a market was that for butter with many producers, many consumers, and no production, distribution, or price control. Demand was believed to introduce negative feedback effect on price and to correct for supply and demand discrepancies.

Work on (electrical circuit) models has indicated that this correcting process may not be as smooth as has been believed. The system can go into oscillation depending on the time lags and other system factors. What has normally been regarded as an equilibrizing force may well tend to operate as an aggravating influence under certain conditions.

Keynesian Theory has it that the negative feedback inherent in an economic system may be too weak to be effective and that Government control has to be introduced in addition. This is another form of negative feedback and it may or may not work, depending on the system constants. Actually, attempts at control by the Government have failed from time to time in the past. The difficulty is always to know whether any action taken has been insufficiently strong to produce a result or whether the result has not yet become apparent.

Model work indicates that delay in applying controls brings about cyclic fluctuations. The post-war economic scene has been characterized by a two-to-three-year cycle and it seems inevitable that this should continue unless remedial controls can be applied within six months or less of the time when the condition

becomes apparent which it is intended to correct. At present it still takes longer than that for such action to be taken.

"*Business Decisions and Forecasting*"—*Professor C. Holt*

PROFESSOR HOLT gave his talk under three headings:

(i) *Sales forecasting.* Quite a lot of work has been done on the question of sales forecasting. Any statistical time series analysis must depend on the assumption that the economic situation remains unchanged but, as is well known, this is not true in real life.

One formula which they have found to give good results was given as:

$$S_{FN} = \cdot 2S + \cdot 8S_F$$

where S_{FN} is the sales forecast for next month;

S the actual sales during this month; and

S_F is the sales forecast that was made for this month.

This is an entirely empirical formula but it has been used to good effect in a number of places.

(ii) *Decision analysis.* Most business decisions hinge on the expectation of sales and sales fluctuations. It is possible to cater for short-term fluctuations in three ways:

(a) to absorb demand peaks with a heavy buffer stock which is replenished during periods of low demand;

(b) to work extra overtime when sales are high and to work short time when sales are low;

(c) to work normal hours only but to engage and discharge labour at short notice as required to follow the level of sales.

Quite apart from the obvious limitations of each of these alternatives, there are different costs associated with doing these different things and in practice most businesses follow a mixed strategy. It is now possible to analyse the factors to be considered in deciding what strategy should be followed and to assist in making this decision on a more rational basis.

(iii) *The use of electronic computers in industry.* Electronic computers can be used to replace clerical labour in carrying out routine clerical tasks and most computer applications are justified by the claim that a computer can do more rapidly, cheaply and reliably what it normally takes large numbers of people to do.

Apart from its ability to do simple sums at a great rate, the computer is specially fitted for carrying out highly complex calculations and its abilities may be put to better use if it is employed on special problems, including decision analysis and forecasting problems. There is a reluctance to concede to the computer the genuine "thinking" capacity which it has; perhaps one should regard computers as able to carry out mental tasks just as one regards, say, the power press as able to do more than a man with a sledgehammer.

"*Sampling and Related Problems*"—*Mr. C. A. Moser*

MR. MOSER spoke on the research the school had done into sampling for sampling surveys. Questions that had been considered included a comparison of professional and amateur interviewers, comparisons of random and quota sampling, and the errors in data collection and interpretation as opposed to the statistical sampling errors.

The comparison between professional and amateur interviewers had been done using two professional market research organizations on the one hand and students from the L.S.E. on the other. Whilst the professional interviewers had got answers to their questions from all but 3 per cent of the people they approached, the L.S.E. students failed to get answers from 14 per cent. There was clearly a technique in getting people to answer a list of questions.

Random sampling was normally preferred by academics because it enables the sampling error to be calculated. Quota sampling is cheaper and quicker and was shown to give accurate results if properly done. Mr. MOSER enlarged on the detailed work that the school have undertaken in building up a formal background to sampling surveys; they have gone a long way to cope with normal conditions and were now looking at the difficulties encountered in more unusual fields, for instance in under-developed countries; it was not easy to construct a basis for a sampling survey of a nomad population where one had only a little information on the number of people in a tribe, or indeed where they were to be found at any given time.

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"*Memory Errors, Volunteer Bias, etc.*"—*Dr. W. A. Belson*

DR. BELSON spoke about the psychological aspects of survey work. They had begun their task about 18 months ago and had started by establishing a bibliography and a statement on the existing state of knowledge in the field. They were now engaged on two pieces of research; the first of these dealt with the memory of those taking part in a survey. Was there any tendency on the part of some people to bring events forward or to push them back, thinking about them in retrospect?

The second project dealt with the effect of bias on the part of volunteers in survey panels, e.g. consumer or listener panels. These panels formed to some extent self-selective groups. Is there anything different about people who accept invitations to sit on panels of this kind? The work done so far has indicated that certain kinds of people are more likely to accept invitations to co-operate in surveys than others. They are usually higher up on the occupational scale, they are more exploratory in outlook and have a wider range of interest and they are more ready to gamble (more of them are subscribers to football pools than those who refuse an invitation); they also tend to be organizers and planners.

People who refuse an invitation to serve on a panel are likely to be more settled in their community (usually having lived there for most of their life), interested in reading, and are more home orientated.

"Programming the Analysis of Sample Surveys"—Mr. A. Stuart

MR. MOSER had referred to the inaccuracies in selecting, recording and interpreting a sample and that these probably exceeded by a fair margin the measurable statistical sampling error. Mr. STUART wound up the afternoon with a statement of the intended work which it was hoped would subject these errors to analysis.

In the course of a brief discussion there was reference to the use of computers in econometric models and to the progress of decision-making techniques. Mr. PURDON asked whether it would be possible to see the L.S.E. hydraulic analogue model of the National Economy and Professor PHILLIPS undertook to demonstrate it after the meeting had closed. A small number of those attending had the working of this model explained by Professor PHILLIPS and were taken through a simulated ten-year period, showing how the model brought out the inter-relation between the various factors affecting the national economy.

Notice

The Proceedings of the Second International Conference on Operational Research, which is being held in Aix-en-Provence from 5–9 September this year, are being published by English Universities Press early in 1961. Orders should be sent to: English Universities Press Ltd., 102 Newgate Street, London, E.C.1.

Notice to Subscribers

Owing to production difficulties it has been necessary to publish Issues 1 and 2 of Vol. 11 in one cover. Any inconvenience to subscribers is regretted.

The Institute of Management Sciences 7th International Meeting

The Institute of Management Sciences announces its 7th International Meeting to be held at the Hotel Roosevelt, New York City, on 20–22 October, 1960. The meeting is intended for administrators and scientists in the field of management, and will include sessions on Behavioral Science and Management Science, Applications and Tools of Management Science and the Use of Computers in Simulation. Registration fees are $15 Members, $20 non-Members and $5 Students. Further information is available from Mr. JAMES TOWNSEND, 30 East 42nd Street, New York 17, New York.

SWORDS

The South Western Operational Research Discussion Society (SWORDS) is affiliated to the Operational Research Society. The Chairman of SWORDS is Mr. G. W. THOMAS, The Steel Company of Wales Limited, Port Talbot, the Honorary Secretary is Mr. J. K. FRIEND, British Resin Products Limited, Penarth, and the Honorary Treasurer is Mr. D. ROBINSON, Richard Thomas and Baldwins Limited, Ebbw Vale.

Notices of SWORDS meeting will be circulated to members of the Society from time to time by the Secretary of the Society, and any members who are in South Wales are welcome to attend those meetings.

SWORDS is the only local society at present affiliated to the Operational Research Society.

Obituary

We have heard with great regret of the recent death of the following members of the Society:—

Dr. Cecil Gordon—Usher Institute of Public Health, who was one of the founder members of the Operational Research Club immediately after the War.

Dr. David Pomeroy—Nuclear Development Corporation of America.

Erratum

Operat. Res. Quart. **10**, No. 4, 192 (1959).

The sentence at the top of the page: "It is interesting to compare these figures with a survey carried out recently amongst the Fellows of the Royal Statistical Society", should follow after Table 1 and immediately before Table 2.

Members of the Operational Research Society

ADDRESS LIST

(as at 31 December, 1959)

IN the following list * indicates full membership of the Society. At 31 December, 1959, the membership figures were 272 full, 288 associate (total 560).

This list has been prepared from the records maintained in the Society's office: any errors should be brought to the attention of the Secretary.

*ABRAMS, Dr. J. W., c/o Canadian National Military Representative, SHAPE, Paris, France.

ABSOLON, C. G., 86 Cardiff Road, Newport, Mon.

*ACKERLIND, Dr. E., P.O. Box 590, Monterey, California, U.S.A.

*ACKOFF, Dr. R. L., Case Institute of Technology, University Circle, Cleveland 6, Ohio, U.S.A.

*ACTON, R. A., Richard Thomas & Baldwins Ltd., O.R. Department, Ebbw Vale, Mon.

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Some Optimization Problems in Advertising Media Planning†

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This is an account of the first stage of a multi-stage research programme in the planning of advertising media schedules. The problems of securing maximum impact and maximum coverage in an intense campaign of short duration for a fixed outlay are discussed. A precise solution of the Impact problem and an approximate solution of the Coverage problem are outlined. These solutions are currently being used, but test results are not yet available. This is, therefore, an interim statement and not a completed case history.

MANUFACTURERS of consumer goods or services usually determine, on grounds more or less *a priori*, the amount of money they believe they can afford to spend on advertising. The decision then facing the advertiser is how to maximize the effect of his advertising; given a fixed sum of money available, what is the most advantageous method of using it?

The criticality of this decision evidently varies from industry to industry, but in the airline case (and, we believe, in other transport industries) not spending more money on advertising than is required to fill the available capacity is of great importance.

For example, let us suppose the case of a flight undertaken daily in the evening from *A* to *B*, primarily for positioning purposes, in order to be able to fly from *B* to *A* the next day. The costings for the latter flight, *B* to *A*, will have already accounted for the need to position and, in order to fill the evening positioning flight at a commercially unattractive time, it is decided to create a special low fare, which will cover direct operating costs and make the positioning flight profitable. A programme of advertising is decided on to publicize the special low evening fare. Now if the aircraft operates at a load factor far below the break-even point, the effect of advertising expenditure will be to increase the loss; if, however, the advertising expenditure is highly successful, enough traffic may be stimulated to create a demand for an extra service, which of course destroys the basis on which the whole operation was predicated. We may leave out of account for this purpose errors in pricing policy, since in

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effect there will be a tendency to overprice and to compensate for this "playing safe" by advertising. Thus it is important to gauge exactly what level of expenditure on advertising is justified.

The advertiser, in making this decision, is confronted with two sets of choices: first, choices between media, involving inter-media comparisons (e.g. whether to spend so much on television, so much in the press, a lesser amount on posters, and so on); and secondly, choices within media (e.g. which newspapers to use and at what frequency of use, and so on). We are here concerned with the second case, namely, to determine which of a group of newspapers should be used and what size of advertisements should be used in order to get the largest possible effect for a given expenditure.

Empirical evidence of the effect of different sizes of advertisement is slender and, since we have it in mind to explore even less well-charted fields, we decided to examine the effect of different assumptions about the way in which advertising works, rather than wait until experimental evidence became available. Like other forms of economic investigation, it is often very expensive and laborious to confirm hypotheses in the field, and we felt it more profitable to build a model, recognizing its axiomatic character, and to judge its validity *ex post facto*.

Two further caveats must be entered. First, we are not concerned here with the contents of an advertisement nor with actual response to an advertisement in the shape of sales or coupon replies: we have not attempted to relate at this stage the seeing of an advertisement to the effect it scores. Secondly, in the case described in more detail below, we are concerned with a number of advertisements in a number of publications, but with the significant condition that each publication carries the advertisement just once. This happens to be a type of campaign which occurs quite frequently in our particular field.

PROBLEM FORMULATION

When we came to specify the problem which we had set ourselves more precisely, we realized that there were two different, but related, problems to be solved. The first of these was to maximize the *coverage* provided by an advertising schedule of given cost. Coverage is defined as the proportion of the population seeing at least one advertisement. The second problem was to maximize total impacts. By *impacts* we mean the number of times an advertisement is seen. This differs from coverage in that a person who sees n advertisements counts only 1 towards coverage, but n towards the total impacts. We decided to attempt to solve both problems. The first step which we took was to inspect the data which were available, and the assumptions which we could make, to formulate functional expressions for these factors.

The readerships of the leading daily and Sunday newspapers and of the leading magazines in Great Britain are available as a result of surveys carried out regularly by the Institute of Practitioners in Advertising[2]. This survey also

gives what are known as "duplication tables". The duplication between newspaper A and magazine B, for example, is the number of people who read both A and B. It is possible to obtain information on higher order "duplications" such as the number of people who read all of any group of four of the publications included in the survey.

Secondly, and obviously, comprehensive cost figures for different sizes of advertisements in different periodicals are available from the publishers.

In addition to the basic factual information provided by these surveys, it was necessary to make certain assumptions regarding the attention value of advertisements. At the time we began this project the best attested relationship in our view appeared to be the square-root rule proposed initially by London Press Exchange Ltd. This says that an advertisement in any periodical which occupies a proportion p of a page is seen by a proportion $\sqrt{p}$ of the readers of that page. In any periodical, however, a whole page is not necessarily seen by all readers. However, for our purposes, we assumed initially that this would be so. In later work we may alter this assumption if it seems necessary.

One further assumption had to be made. Suppose that the attention value of a whole page in publication A_1 was a_1. Then the square-root rule says that the attention value of an advertisement occupying a proportion p_1 of a page in A_1 would be $a_1\sqrt{p_1}$. Similarly, for a publication A_2 with advertisement size p_2, the attention value would be $a_2\sqrt{p_2}$. But A_1 and A_2 duplicate, and the duplicated readership of A_1 and A_2 is, say, a_{12}. The question to be resolved is the proportion of the population who see both an advertisement of size p_1 in A_1 and p_2 in A_2. We assumed that there would be no predisposition for someone who had seen the advertisement in A_1 to see it also in A_2 or not to see it in A_2, and that the answer to this question would be

$$a_{12}\sqrt{p_1}\sqrt{p_2}.$$

This is the simplest assumption to make. If, however, it were proved that a person who reads both A_1 and A_2, and who had seen an advertisement in A_1, was more or less likely to see an advertisement in A_2 than another reader of both who had not seen an advertisement in A_2, then the answer would take some other form, say $ra_{12}\sqrt{p_1}\sqrt{p_2}$, where r is a correlation coefficient of some type. Evidence in support of any hypothesis is very sparse.

However, by considering the various combinations of publications which are possible, these hypotheses enabled us to write down a general expression for the proportion of the population (e.g. all adults, or all housewives, or all AB social group—the a's being chosen accordingly) under consideration seeing j advertisements, that is receiving j impacts given that advertisements have appeared in k publications. This expression is

$$I_j = \sum_{r=j}^{k} (-1)^{r+j}\binom{r}{j} H_r \qquad (1)$$

where I_j is the proportion of the population receiving j impacts and

$$H_r = \sum_{u_r > u_{r-1}}^{k} \sum_{u_{r-1} > u_{r-2}}^{k} \cdots \sum_{u_1 = 1}^{k} a_{u_1 u_2 \cdots u_r} z_{u_1} z_{u_2} \cdots z_{u_r}$$

with $z_i = \sqrt{p_i}$.

For two publications, for example, one has (introducing I_0 also, with obvious meaning):

No. of advertisements seen (x)	Proportion of people seeing x advertisements	Symbol
0 advertisements	$1 - a_1 z_1 - a_2 z_2 + a_{12} z_1 z_2$	I_0
1 advertisement	$a_1 z_1 + a_2 z_2 - 2a_{12} z_1 z_2$	I_1
2 advertisements	$a_{12} z_1 z_2$	I_2

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It is convenient to introduce identity labels w_j to attach to the expression for j impacts. Then, if the number of people receiving j impacts is denoted by I_j, one can define an expression E, which will be called the *effectiveness index*, thus

$$E = w_0 I_0 + w_1 I_1 + \ldots + w_k I_k.$$

The interpretation to be put on the effectiveness index E varies according to the values chosen for the w_j. For example, if $w_0 = 0$ and $w_i = 1$ for $i = 1, 2, \ldots, k$, then it is equal to the coverage; that is, it is a measure of the proportion of the population seeing at least one advertisement. If, however, w_j were put equal to j, where j takes the values $0, 1, 2, \ldots, k$, etc., then the effectiveness index would become exactly equivalent to the mean number of impacts.

In the work at present being described, we were interested in both factors: the mean number of impacts per person, denoted by $F(z_1, z_2, \ldots, z_k)$, and the coverage, denoted by $G(z_1, z_2, \ldots, z_k)$. Functional expressions for these can be derived without difficulty by using the quantities H_r. In fact,

$$F(z_1, z_2, \ldots, z_k) = H_1 = \sum_{i=1}^{k} a_i z_i \tag{2}$$

and

$$G(z_1, z_2, \ldots, z_k) = \sum_{j=1}^{k} (-1)^{j-1} H_j. \tag{3}$$

The problems which we had to consider, therefore, were special cases of one general problem. This was to maximize the effectiveness index, given the values of the w_j subject to a number of conditions on the values of the z's. As each z is a proportion, it is clear that all z's must lie between zero and unity. Furthermore, as the total sum of money available for advertising purposes is usually limited to, say, £c, and as the costs of full-page advertisements are known to be £c_1, £c_2, ... and so on, there is a cost condition on the z's also.

The conditions may be summarized thus:

Nature of Restriction	*Mathematical Expression*
Proportionality of z's (no advertisement is less than zero or more than unity as proportion of a page)	$0 \leqslant z_i \leqslant 1 \ (i = 1, 2, \ldots, k)$
Cost (the total sum spent must equal £c)	$\sum_{i=1}^{k} c_i z_i^2 = c$
(The problem is non-trivial)	$\sum_{i=1}^{k} c_i \geqslant c.$

The method of solution of problems of this type is well established and need not be described. A good, short account is given by Saaty[1]. The difficulty of obtaining solutions in particular cases depends upon *the exact form of the function E to be maximized.*

THE MAXIMIZATION OF IMPACTS

The problem can be solved in fairly simple algebraic terms when the function E being maximized is, in fact, F, the mean number of impacts. In this case the solution is given by carrying out the iterative calculations outlined below.

Solution of impacts maximization problem:

(1) Compute

$$x_i = (ca_i^2/c_i^2)\left\{\sum_{i=1}^{k} (a_i^2/c_i)\right\}^{-1} \quad (i = 1, z, \ldots, k).$$

(2) Arrange x_i in ascending order of magnitude and re-label as x_i (re-label z_i correspondingly).

(3) If no $x_i \geqslant 1$, then $z_i^2 = x_i$;

If $x_i < 1$ for $i = 1, 2, \ldots, (k-a)$,

$x_i \geqslant 1$ for $i = (k-a+1), \ldots, k$.

Then $z_i^2 = 1$ for $i = (k-a+1), \ldots, k$,

and compute

$$u_i = \left(c - \sum_{i=k-a+1}^{k} c_i\right)(a_i^2/c_i^2)\left\{\sum_{i=1}^{k-a} (a_i^2/c_i)\right\}^{-1} \quad \{i = 1, 2, \ldots, (k-a)\}.$$

(4) Go back to (2), operating on u_i as formerly on x_i.

The meaning of this result is that to maximize impacts when advertising in a number of publications, advertising space should be bought in them in inverse proportion to the square of the cost per thousand readers, except that the maximum permissible size of advertisement in any one publication is a whole page.

THE MAXIMIZATION OF COVERAGE

When the function to be maximized is the coverage G, however, the problem cannot be solved in simple algebraic terms except in the case $k = 2$, that is, when only two publications are involved. Furthermore, the number of terms in the expression for G is $2^k - 1$, and in practical cases, when 10 or more publications may be under consideration, the sheer size of the equations (4) becomes alarming. Even the evaluation of G in such cases would be a lengthy process.

However, an approximation to G was suggested by an examination of IPA data. This was to write $a_{ij} = a_i a_j$, $a_{ijl} = a_i a_j a_l$, and so on. We plotted

$$x_{ij} = a_{ij} - a_i a_j,$$

$$x_{ijl} = a_{ijl} - a_i a_j a_l,$$

for the same group of 15 periodicals mentioned above, and obtained histograms which suggested that *in the case of publications suitable for this type of advertising* we could write

$$a_{ijl} \ldots p \sim a_i a_j \ldots a_p,$$

and it is apparent that in this case,

$$G \sim \left\{1 - \prod_{i=1}^{k} (1 - a_i z_i)\right\}. \tag{4}$$

This convenient-looking expression is, in fact, useful for calculating the coverage obtained, given values for the z_i. But it is difficult to use in determining what the optimal z_i are.

It seemed necessary, therefore, to find some other form of approximation—albeit cruder—to the coverage expression which could be used in practical applications. The obvious course of action was suggested by making the first approximation rather more general. Further examination of IPA figures showed that if

$$\bar{a} = \frac{1}{k} \sum_{r=1}^{k} a_r = a, \quad \text{say,}$$

in the case of 15 periodicals considered by us as most suitable for a certain campaign, then

$$\bar{a}_{..} = \frac{1}{k(k-1)} \sum_{j=i+1}^{k} \sum_{i=1}^{k} a_{ij} \sim a^2,$$

$$\bar{a}_{...} = \frac{1}{k(k-1)(k-2)} \sum_{(ij+1)}^{k} \sum_{j=i+1}^{k} \sum_{i=1}^{k} a_{ijl} \sim a^3.$$

In fact, we found that $\bar{a} - a^2 = 0{\cdot}0002,$

$$\bar{a} - a^3 = -0{\cdot}0002.$$

We supposed, on the basis of this evidence, that $\bar{a}_n \sim a^n$ for all $n = 1, 2, \ldots, k$, so long as k was reasonably large (say $k \geqslant 10$). Consequently, if for the sake of seeing where it would lead, we assumed that the advertisements in the various publications were chosen to have equal attention values z, then the coverage would be, roughly,

$$G^* = \{1 - (1 - az)^k\}. \tag{6}$$

Remembering that what was desired was to reduce the number of terms to be handled in the coverage expression, we considered the quality of approximation to G^* given by taking the sum of terms up to and including those of order r in (az), by examining the *relative difference*

$$G_r^*/G^*$$

where

$$G_r^* = \sum_{i=r+1}^{k} (-1)^{i-1} \binom{k}{i} a^i. \tag{7}$$

This relative difference was tabulated for various values of a, z, k and r, and the results showed that in the region of values of a, z and k of interest to us, the proportionate error would not exceed 10 per cent for $r = 3$. In other words, a reasonably satisfactory approximation could be obtained to the coverage expression by truncating it after third-order terms in the z's.

We realized that this was a far from rigorous procedure, and as we have described it, it represents a written equivalent to thinking aloud. But such approximate analyses are very useful in operational research and, according to Polya, are indispensable in most mathematical work.

Thus we decided that we should consider the approximate form

$$G' = H_1 - H_2 + H_3.$$

Even so, G' would be a very large expression, and still too awkward to allow the equations (4) to (7) to be solved in simple algebraic terms. An iterative type of calculation, an extension of Newton's method for finding the zeros of functions by means of successive approximations, was employed. This method also is described by Saaty[1].

The calculation was programmed for an IBM 650 computer, and a number of optimizations were carried out, using as the initial trial solution in every case the solution to the corresponding impacts maximization problem. We noticed at once that the differences between the initial and final solutions were in many cases very small. In the course of a series of 12 optimizations, each involving 10 publications, only 1 in 4 of final values of z's differed from the initial values by a factor of more than 5 per cent. This made us wonder whether by merely solving the fairly easy impacts problem we could not obtain, almost always, a satisfactory, though not necessarily absolutely optimal, solution to the coverage problem.

It seemed desirable to establish the conditions under which the solutions to the two problems would, in fact, be identical, and after a little time we managed to prove the following theorem:

Theorem. The set of values z_i' $(i = 1, 2, \ldots, k)$ which maximize

$$\sum_{i=1}^{k} a_i z_i$$

subject to the conditions $\sum_{i=1}^{k} c_i z_i^2 = C$

and $0 \leqslant z_i \leqslant 1 \quad (i = 1, 2, \ldots, k),$

also maximizes $G = \sum_{r=1}^{k} (-1)^{r-1} H_r$

subject to these conditions if

(i) $$a_{u_1 u_2 \ldots u_j} = \prod_{r=1}^{j} a_{u_r} \quad (j = 1, 2, \ldots, k),$$

and if

(ii) $$(a_i^2/c_i) = \lambda \qquad (i = 1, 2, \ldots, k),$$

where λ is some constant.

It had been demonstrated previously that to secure the maximum number of impacts, advertising space should be bought in inverse proportion to the square of the cost per thousand readers of each publication. As, however, the solutions to the coverage and impacts maximization problems are identical under the conditions of the above theorem, we had in fact verified the rule:

Rule. When, for any group of publications,

(i) $$a_{u_1 u_2 \cdots u_j} = \prod_{r=1}^{j} a_{u_r} \quad (j = 1, 2, \ldots, k),$$

and if

(ii) $$(a_i^2/c_i) = \lambda \qquad (i = 1, 2, \ldots, k).$$

> The optimum schedule is obtained by both as regards coverage and impacts by buying space in inverse proportion to the square of the cost per thousand for each publication, subject to the qualification that no advertisement should occupy more than one whole page.

We had already shown that the first condition for the theorem to be valid was satisfied, approximately, by most publications of interest to us. The next step was to see how well the second condition was satisfied. The values for

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15 publications are given below, ranked in ascending order:

i	(a_i^2/c_i)	i	(a_i^2/c_i)
1	0·01	9	0·03
2	0·01	10	0·03
3	0·01	11	0·03
4	0·02	12	0·04
5	0·02	13	0·05
6	0·02	14	0·11
7	0·02	15	0·14
8	0·02		

The variation appeared somewhat large, and we examined the "tail" values from 0·04 upwards to see if these publications numbers 12–15 were in any sense different in type from the others. It would be impolitic to divulge the identities of these publications, or to enable them to be identified. It must suffice to say that numbers 14 and 15 were high-cost, high-prestige publications, whereas numbers 12 and 13 were different in make-up to the other publications in having more, but smaller, pages.

Our experience has been that variability such as that existing among the (a^2/c) values for periodicals 1–11 is not too great to prevent the application of the theorem, and even when publications with larger values, such as numbers 12 and 13, were included in iterative calculations, the initial and final answers rarely differed by much.

Consequently, the procedure which we have adopted to determine accurately the allocation of money to maximize impacts, and determine approximately but quickly the allocation to maximize coverage, is as follows. First, if the information is not already available, we check that the conditions of the theorem are satisfied, at least to a reasonable approximation. Secondly, we obtain the values $(z_1^0, z_2^0, \ldots, z_k^0)$ which maximize impacts. Finally, we calculate the corresponding (approximately maximal) coverage

$$G = \left\{1 - \prod_{i=1}^{k} (1 - a_i z_i^0)\right\}.$$

This method, which is suitable for our use, is not of course so in general, as the necessary conditions are not always satisfactorily met.

DISCUSSION AND CONCLUSIONS

The optimization procedure which we have developed, it is plain, does not in general produce an advertising schedule which maximizes coverage. The resulting schedule will, within the limits of our assumptions, produce maximum total impacts and, probably, a high level of coverage. Generally speaking, total impacts and coverage cannot be simultaneously maximized for a fixed cost.

The difficulties in solving some of these problems are computational rather than mathematical, and our method has been to develop approximate working rules. To do this we have not found it necessary to use any out-of-the-way methods. For this reason, mathematical derivations have been deliberately omitted from this paper. It is hoped at a later date, when some results of applying these methods are to hand (and practical applications are being made now), to publish a more detailed account, which will for completeness include mathematical proofs where required.

We think it proper to mention what we propose to do at the next stage of our investigations. First, we propose to take into account the effect of a series of advertisements in one or more publications; this will display, we intend, the effect of decay and rejuvenation throughout a series of advertisements. Secondly, we propose to carry out a similar investigation for television advertising and to combine both into an inter-media comparison where both the press and television are used simultaneously. Thirdly, we hope to investigate the effect of varying intervals between advertisement appearances, both within one medium and in the inter-media situation. We are, however, still in the contemplatory stage at the moment.

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An Algorithm for the Allocation Problem*

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This paper* sets out a procedure for solving allocation problems, on different lines from procedures based on linear programming.

IN "A Combinatorial Algorithm"[1] the author posed and solved the following problem: let us suppose we are given a square $(n \times n)$ array of real numbers $\{a_{ij}\}$ $(i,j = 1, \ldots, n)$, and let it be required to find, among all sums $\sum_{i=1}^{n} a_{ir_i}$ in which r_i is the ith member of a permutation of the numbers $1, \ldots, n$, that subset whose members have the least value. (For brevity, this subset will be called the set of minimum sums.) It will be seen that this is, in fact, the allocation problem. If there are n activities that must be carried out at n locations, and a_{ij} is the cost of carrying out activity i at location j, then the set of minimum sums gives us the set of cheapest ways of allocating the activities to the locations.

(2) Let us call the least elements in each row the *row-minima*. It is obvious that *if* the row-minima are so disposed that we can choose one from each row, no two being in the same column, then such a set is a minimum sum; and, moreover, that in this case only such a set of row-minima can be a minimum sum. Thus, for example, in the array

$$\begin{matrix} 1 & 1 & 3 \\ 4 & 1 & 1 \\ 1 & 5 & 1 \end{matrix}$$

the sums $(a_{11}+a_{22}+a_{33}), (a_{12}+a_{23}+a_{31})$ consist entirely of 1's. Any other sum of elements, no two of which lie in the same row or column, will contain elements greater than 1, but none less, and therefore will not be in our required set.

(3) In general, however, this condition will not be satisfied. It is, however, possible to derive from our original array $\{a_{ij}\}$ another, $\{\hat{a}_{ij}\}$, with the properties:

(i) Σa_{ir_i} belongs to the required set of minimum sums if and only if $\Sigma \hat{a}_{ir}$ belongs to the set of minimum sums of the array $\{\hat{a}_{ij}\}$;

* The basis of this paper is one published by me in 1946, which gave an algorithm for allocation problems. The circumstances of the time did not allow the results to be used then. I am indebted to Dr. Vajda for the suggestion that the main results should be published in a place and under a title where they are more likely to be found by readers to whom they might be of interest. At the same time I have taken the opportunity to fill a gap and make a minor correction to the original paper.

The algorithm does not seem to lead to less laborious computation than the better known methods; on the other hand it seems to me closer in spirit to the essentially combinatorial nature of the problem.—T. E. Easterfield.

(ii) it is possible to choose from among the row-minima of $\{\hat{a}_{ij}\}$ a set of n no two of which lie in the same row or column;

(iii) any row-minimum of $\{\hat{a}_{ij}\}$ lies in at least one minimum sum.

In fact, our derived array $\{\hat{a}_{ij}\}$ will enable us to find all permutations r_i such that Σa_{ir_i} is a minimum sum of the original array.

(4) $\{\hat{a}_{ij}\}$ is formed from $\{a_{ij}\}$ by a series of steps, in each of which a constant quantity is subtracted from every element of one or more columns. Such a step will alter the pattern of row-minima; but since each column contributes one term to any sum, all sums will be altered by the same amount, and the permutations giving rise to minimum sums will be unchanged.

(5) We wish to find a pattern of row-minima such that we can choose n of them, one lying in each row and one in each column. It is, however, easier to work with another condition on the pattern that may be shown to be equivalent to this. Let us define a set of $r(\leqslant n)$ columns as being *adjusted* if each subset of $s(\leqslant r)$ columns in it contains row-minima from *at least* s rows. If an adjusted set of r columns contains row-minima from just r rows, the set will be called *exactly adjusted.*

(6) The paper proves as theorem I: A necessary and sufficient condition for it to be possible to choose one row-minimum from each column of a set of columns in such a way that no two of these lie in the same row is that the set be adjusted. Further, if the set contains no exactly adjusted subset, we may include any one of its row-minima, arbitrarily chosen, in such a choice.

(7) It is then proved that the following algorithm leads to the whole array being adjusted:

Let us represent the ith column of the array by (i), the set of the ith, jth and kth columns by (ijk) (where $i<j<k$), and so on; and let us order all sets of columns as follows: (1) (2) (12) (3) (13) (23) (4) (14) (24) (124) (34) (134) (234) (1234) .... (Formally $(a_1 a_2 \ldots a_r)$ precedes $(b_1 b_2 \ldots b_s)$ if $a_r < b_s$, *or* if $a_r = b_s$ and $(a_1 \ldots a_{r-1})$ precedes $(b_1 \ldots b_{s-1})$. The null set is taken as preceding every other.) We then adjust each set in order.

(8) First, if (1) contains no row-minimum, we subtract from every element of (1) the least number that will make it contain one. Next, treat (2) likewise. Now if (12) contains row-minima in one row only (through their both being in the same row), subtract from every element of both columns the least number required to introduce a row-minimum of another row into (12). Now (12) contains row-minima in two rows, hence it is adjusted. Go on similarly, subtracting from every element of the set reached the least number required to adjust it. The proof in the paper shows that with the given ordering of the sets of columns, the adjustment of any set remains unspoiled by all later steps. Thus we come eventually to a state in which the whole array is adjusted.

(9) Finally, if any subset of columns is exactly adjusted, the subtraction of a very small quantity from every element in the subset will remove from all other columns row-minima in the rows in which the subset has row minima,

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but no other row-minima. If this procedure is applied to every exactly adjusted subset left in the final array, we end with a pattern of row-minima such that every row-minimum is in an exactly adjusted set of columns that has no exactly adjusted subset, and hence by theorem I, will appear in at least one minimum sum.

(10) The proof that this algorithm does in fact give the desired answer makes it clear that the ordering of all subsets of columns described in paragraph 7 above is not the only possible one. The essential condition on the ordering is that, if $(a_1 \dots a_r)$ precedes $(b_1 \dots b_s)$, the adjustment of the latter will not spoil that of the former if we have already ensured the adjustment of one of the sets of columns containing every column in each set bar one. (For example, adjusting (145) will not spoil the adjustment of (123) provided that one of (1234) (1235) (1345) and (2345) precedes (145).) Further, any set must be preceded by all its proper subsets. (In the paper, only the latter condition was given.)

(11) For a large array, the number of steps involved is very large. If the job is being done by hand, however, a little practice will allow the worker to take short cuts in choosing sets of columns which will most quickly lead to the final stage.

(12) As an example we give the working on a 6×6 array:

9	22	58	11	19	27
43	78	72	50	63	48
41	28	91	37	45	33
74	42	27	49	39	32
36	11	57	22	25	18
3	56	53	31	17	28

(Note: We have underlined all row-minima in this and the successive derived arrays.)

Here all sets of columns up to (123) are adjusted. Subtract 2 from each element of (4) and obtain

9	22	58	9	19	27
43	78	72	48	63	48
41	28	91	35	45	33
74	42	27	47	39	32
36	11	57	20	25	18
3	56	53	29	17	28

All sets of columns up to (1234) are adjusted. Subtract 10 from each element of (5) and obtain

9	22	58	9	9	27
43	78	72	48	53	48
41	28	91	35	35	33
74	42	27	47	29	32
36	11	57	20	15	18
3	56	53	29	7	28

(15), (25), (125), (35), (135), (235) and (1235) are adjusted; (45) is not. Subtract 2 from each element of each of these columns and obtain

9	22	58	7	7	27
43	78	72	46	51	48
41	28	91	33	33	33
74	42	27	45	27	32
36	11	57	18	13	18
3	56	53	27	5	28

(145), (245), (1245) are adjusted; (345) is not. Subtract 2 from each element of all three columns and obtain

9	22	56	5	5	27
43	78	70	44	49	48
41	28	89	31	31	33
74	42	25	43	25	32
36	11	55	16	11	18
3	56	51	25	3	28

Every set up to (12345) is now adjusted; (6) is not. Subtract 5 from each element of (6) and obtain

9	22	56	5	5	22
43	78	70	44	49	43
41	28	89	31	31	28
74	42	25	43	25	27
36	11	55	16	11	13
3	56	51	25	3	23

The whole set is now adjusted. We may apply the process of paragraph 9 which we may exhibit by subtracting 1 from each element of columns 3 and 4 and obtain

$$\begin{array}{cccccc} 9 & 22 & 55 & 4 & 5 & 22 \\ \underline{43} & 78 & 69 & 43 & 49 & \underline{43} \\ 41 & \underline{28} & 88 & 30 & 31 & \underline{28} \\ 74 & 42 & \underline{24} & 42 & 25 & 27 \\ 36 & \underline{11} & 54 & 15 & \underline{11} & 13 \\ \underline{3} & 56 & 50 & 24 & \underline{3} & 23 \end{array}$$

The sets of columns (3), (4) and (1256) are all exactly adjusted but contain no exactly adjusted subset.

It will be found that there are just two minimum sums,

$$a_{21}+a_{52}+a_{43}+a_{14}+a_{65}+a_{36} \quad \text{and} \quad a_{61}+a_{32}+a_{43}+a_{14}+a_{55}+a_{26},$$

which are, in the original,

$$43+11+27+11+17+33=142 \quad \text{and} \quad 3+28+27+11+25+48=142.$$

(13) It will be seen that although each row-minimum left in the final array appears in at least one minimum sum, only a limited number of combinations of row-minima is possible. The following process allows all possible combinations to be found.

(14) We may suppose that the final array has no exactly adjusted subsets, for if not we can treat any exactly adjusted subset independently of the rest of the array.

(15) If we choose any particular row-minimum, we know that it will occur in at least one minimum sum. Moreover, the minimum sums in which it occurs are those of the array formed by suppressing the row and the column in which it lies. This array must obviously be adjusted; but it may now contain an exactly adjusted subset, necessitating the suppression of some row minima by the method of paragraph 9 and the reduction of the array to a set of exactly adjusted sets, none of which contains an exactly adjusted subset.

(16) For a small array, the identification of exactly adjusted subsets can be done easily by inspection. If a mechanical procedure is needed, then for any set of r columns let us define its *excess* as $(s-r)$, where s is the number of rows in which it contains row-minima. An exactly adjusted set of columns has excess zero. Note the excess for each set of columns, at the end of the calculation of paragraphs 8 and 9. (It would probably save no time to note the excess of each column treated during this calculation, since the excess of a set of columns that has been adjusted may change as later sets of columns are dealt

with. We know that it cannot fall below zero; but it may rise, and later it may also fall.)

(17) When a row and a column are removed, as in paragraph 15, any other column will lose a row-minimum if it contained one in the suppressed row, but not otherwise. Thus for any set of columns in the remaining array, the excess will fall by one if *any* column of the set had a row-minimum in the suppressed row, but otherwise be unchanged. It is now easy to see which sets of columns are exactly adjusted, which row-minima must be suppressed, as in paragraph 9, and thus where to start from for the next step.

(18) All minimum sums may therefore be found by the following process:

Break up the array found by the procedure of paragraphs 8 and 9 into exactly adjusted sets of columns without exactly adjusted subsets. Deal with each of these separately.

Take the first row-minimum in column 1 of the first one of these sets, delete the row and column in which it occurs, and break up the remaining array similarly into minimal exactly adjusted sets of columns.

Take the first row-minimum in column 1 of the first of these, and proceed as before.

When this procedure comes to a stop, go back to the last column from which the first row-minimum was taken, and take the next, and proceed as before.

Proceed similarly, going back to the most recent stage at which an alternative choice was possible, until at last all possibilities have been exhausted.

(19) Thus in the example, the columns 3 and 4 are each exactly adjusted. If we delete these and the corresponding rows, and write in only those elements where row-minima occur in the remaining array, we get

$$\begin{array}{cccc} a_{21} & . & . & a_{26} \\ . & a_{32} & . & a_{36} \\ . & a_{52} & a_{55} & . \\ a_{61} & . & a_{65} & . \end{array}$$

(It will be seen that we can work here with the pattern formed by the row-minima only; the numbers they stand for no longer affect the working.)

If we choose a_{21}, the first row and first column must be deleted; a_{36} is now the sole row-minimum in the last column, so a_{32} must be suppressed; a_{52} is the only remaining row-minimum in the first remaining column, so a_{55} must be suppressed. Thus the only minimum sum containing a_{21} is

$$a_{14}+a_{21}+a_{36}+a_{43}+a_{52}+a_{65}.$$

The only point to which we can go back to make a different choice is the beginning: instead, we can choose a_{61}. If we delete column 1 and row 6, the only row-minimum in column 5 is a_{55}; hence a_{52} must be suppressed, and, in

consequence, a_{36}. Thus the only minimum sum containing a_{61} is

$$a_{14}+a_{26}+a_{32}+a_{43}+a_{55}+a_{61}.$$

Since there are no other row-minima in column 1, there are no other minimum sums.

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Stock Control with Random Opportunities for Replenishment

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In most stock control systems, stock is replenished by re-ordering either at regular intervals or upon depletion below an agreed level. Situations sometimes arise, however, in which replenishment tends to be governed by the incidence of external opportunities which occur at irregular intervals, usually as a result of a complex schedule of production or distribution. Conventional stock control procedures can then become difficult to operate effectively; this article discusses an alternative procedure which can be used in such circumstances, and shows how optimum control levels may be calculated.

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INTRODUCTION

IT is in the interests of every stock controller that he should be able to replenish his stocks immediately on demand, or at any rate within a guaranteed lead time. In this respect, he is sometimes liable to come into conflict with the interests of his supplier, who may have to fit the replenishment order into a complex schedule of production or distribution. Where the supplier and the stock controller form part of the same organization—as with a production unit and its finished goods store, or a central warehouse and its local depot—the objective must be to resolve this conflict of interests to the maximum economic benefit of the organization as a whole. If the opportunities for economical stock replenishment are irregular and comparatively infrequent, conventional stock control procedures may become inefficient in the light of this objective. An alternative system, which allows operating costs to be minimized provided certain general conditions apply, is outlined below.

The system permits two grades of replenishment: "opportunity" replenishment, available only during specific opportunities to be notified by the supplier, and "emergency" replenishment, bearing a much higher cost but available at short notice if stock should become exhausted between opportunities. It is assumed that the controller need not place an order for opportunity replenishment until he is informed that an opportunity is imminent, releasing him from the need to keep safety stock to protect against uncertainty in lead time.

In manufacture, opportunities may reflect particular machine settings or natural breaks in the continuity of production: in distribution, they may involve taking advantage of scheduled sailings or lorry routings between the locations of the supplier and the stockholder. A necessary condition for the successful operation of this type of stock control system is that the incidence of opportunities should be independent of demand for the item concerned; for this reason, it can most profitably be applied to items which constitute only a small proportion of the supplier's total commitments—so that knowledge

of the controller's requirements is not critical to the supplier's advance planning —yet which have a substantial nuisance value in the high costs involved if the supplier has to meet an order at an inopportune point in his schedule.

The analysis that follows shows how stock control levels can be determined which will balance the frequencies of "opportunity" and "emergency" replenishment so as to minimize overall cost, first on the assumption that opportunity replenishment carries no set-up cost, and then on the alternative assumption that a finite cost is involved which is less than the cost of emergency replenishment. The analysis depends for its simplicity on assumptions of randomness in the incidence of opportunities and of unit issues from stock; the effect on the theory of departures from these conditions is discussed at the end of the article.

REPRESENTATION AS A BINARY SEQUENCE

It will be assumed that opportunities for replenishment and unit issues from stock occur at random in accordance with Poisson distributions with average frequencies λ_0 and λ_1 per unit time respectively. This makes it possible to regard opportunities (0) and issues (1) as forming a random binary sequence

100101100001011 ...

where the probability that an event is an issue is $p \equiv \lambda_1/(\lambda_0+\lambda_1)$ and the probability that it is an opportunity is $1-p \equiv \lambda_0/(\lambda_0+\lambda_1)$. The level of stock at any instant will then be determined by the sequence of events up to that instant; the time intervals between successive events, which form a random sample from a negative exponential distribution with mean $1/(\lambda_0+\lambda_1)$, are of no significance in this context. In fact, only one of the parameters required to calculate optimum replenishment levels—the cost of holding unit quantity of stock between one event and the next—is a function of time and this can be replaced, without any effect on the model, by a constant h representing the cost of holding a unit of stock over the average time $1/(\lambda_0+\lambda_1)$ between successive events.

SOLUTION FOR ZERO COST OF OPPORTUNITY REPLENISHMENT

If replenishment during an opportunity involves no set-up cost, nothing will be lost by replenishing stock to its maximum level of M units every time an opportunity occurs. It is assumed that stock is also replenished to this level if it becomes exhausted between opportunities, and that such an emergency replenishment bears a set-up cost k.

The state of the system can be defined as the number N of units in stock, taking any of the values $1, 2, \ldots, M$. At each opportunity, stock is immediately replenished to M; at each issue, stock is depleted by one unit, unless the

initial state was $N = 1$, in which case emergency replenishment would immediately raise it to M. This defines a simple Markov process which can be set out as follows:

No. of units in stock before event	No. of units in stock after event	
	(i) if event is issue (probability p)	(ii) if event is opportunity (probability $1-p$)
M	$M-1$	M
$M-1$	$M-2$	M
$M-2$	$M-3$	M
...	...	...
...	...	...
3	2	M
2	1	M
1	M	M

A stock of N units, where $N \neq M$, implies that the last event was an issue and that the stock was previously $N+1$ units. This gives the following relations for the state probabilities P_N:

$$P_{M-1} = pP_M,$$
$$P_{M-2} = pP_{M-1} = p^2 P_M,$$
$$P_{M-3} = pP_{M-2} = p^3 P_M,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$P_1 \quad = pP_2 = p^{M-1} P_M.$$

It follows that

$$1 \equiv P_1 + P_2 + \ldots + P_{M-1} + P_M = (p^{M-1} + p^{M-2} + \ldots + p + 1).P_M = \frac{1-p^M}{1-p}.P_M,$$

so that

$$P_M = \frac{1-p}{1-p^M}$$

and, generally,

$$P_N = p^{M-N}.\frac{1-p}{1-p^M}.$$

The average number of units in stock will be

$$\sum_{N=1}^{M} N.P_N = \frac{1-p}{1-p^M}.[1.p^{M-1} + 2.p^{M-2} + \ldots + (M-1).p + M.1],$$

which reduces to

$$\frac{M}{1-p^M} - \frac{p}{1-p}.$$

The average cost of stockholding between consecutive events can then be obtained by multiplying this expression by h, the unit cost of stockholding over the average time between events. Against this cost must be balanced the

cost k of emergency replenishment whenever stock is exhausted between opportunities. This cost will arise only when stock is given by $N = 1$ and the next event is an issue, a situation which occurs with probability

$$pP_1 \equiv \frac{p^M(1-p)}{1-p^M}.$$

The total expected cost over the average interval between consecutive events then becomes

$$C(M) = \frac{hM}{1-p^M} - \frac{hp}{1-p} + k.\frac{p^M(1-p)}{1-p^M}.$$

Ignoring for the moment the limitation of M to integral values, the optimum level of M is given by

$$0 = \frac{dC}{dM} = \frac{h}{1-p^M} + \frac{hM}{(1-p^M)^2}.p^M \log_e p + k.\frac{(1-p)p^M \log_e p}{(1-p^M)^2}.$$

Putting $\log_e p = -l$, so that $p^M = e^{-lM}$,

this gives $$h(1-e^{-lM}) = [hM + k(1-p)]le^{-lM},$$

i.e. $$l.\left[M + \frac{k}{h}(1-p)\right] = e^{lM} - 1.$$

This equation for M can be solved without difficulty by numerical or graphical methods; it is essentially of the form $X = Ae^X$ where A represents $e^{-k/h(1-p)l-1}$ and the unknown variable X represents $l.[M + k/h(1-p)] + 1$. The optimum integral value of M will be the next integer above or below the value given by the solution of this equation.

SOLUTION FOR FINITE COST OF OPPORTUNITY REPLENISHMENT

It will now be assumed that opportunity replenishment carries a finite set-up cost k_0, less than the cost k of emergency replenishment. In these circumstances it may no longer pay to replenish stocks every time an opportunity occurs, particularly if the quantity needed to replenish to maximum is only small. The logical method of operation will then be to take advantage of an opportunity to replenish stock only if the stock is at or below some minimum level L. This generates a Markov process which can be set out as in the table on page 134.

The states $N = L, L-1, \ldots, 2, 1$ can, as before, only arise if the last event was an issue, so that

$$P_L = pP_{L+1},$$
$$P_{L-1} = pP_L = p^2 P_{L+1},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$P_1 = pP_2 = p^L P_{L+1}.$$

No. of units in stock before event	No. of units in stock after event	
	(i) if event is issue (probability p)	(ii) if event is opportunity (probability $1-p$)
M	$M-1$	M
$M-1$	$M-2$	$M-1$
$M-2$	$M-3$	$M-2$
...	...	...
$L+2$	$L+1$	$L+2$
$L+1$	L	$L+1$
L	$L-1$	M
$L-1$	$L-2$	M
...	...	...
3	2	M
2	1	M
1	M	M

On the other hand, the states $N = M-1, M-2, \ldots, L+1$ can arise either from the state $N+1$ followed by an issue or from the state N followed by an opportunity, giving equations

$$P_N = pP_{N+1} + (1-p)P_N,$$

i.e. $P_N = P_{N+1}$, so that

$$P_{L+1} = P_{L+2} = \ldots = P_{M-1} = P_M.$$

Adding probabilities for all N,

$$1 \equiv \sum_{N=1}^{M} P_N = (p^L + p^{L-1} + \ldots + p^2 + p + M - L).P_M = \left[M - L + \frac{p(1-p^L)}{1-p}\right] P_M,$$

so that

$$P_M = \left[M - L + \frac{p(1-p^L)}{1-p}\right]^{-1}.$$

The average number of units in stock is given by

$$\Sigma N.P_N = \left[M - L + \frac{p(1-p^L)}{1-p}\right]^{-1} \times [1.p^L + 2.p^{L-1} + \ldots + L.p + (L+1) + (L+2) + \ldots + M]$$

$$= \left[M - L + \frac{p(1-p^L)}{1-p}\right]^{-1} \cdot \left[\frac{p}{1-p}\left\{L - \frac{p(1-p^L)}{1-p}\right\} + \frac{M(M+1)}{2} - \frac{L(L+1)}{2}\right].$$

The cost of stockholding between consecutive events is then obtained by multiplying by h as before. The cost of replenishment will be composed of k multiplied by the probability $pP_1 \equiv p^{L+1}P_M$ of emergency replenishment, plus k_0 multiplied by the probability of opportunity replenishment, which is

$$(1-p)(P_1 + P_2 + \ldots + P_L), \quad \text{i.e.} \quad p(1-p^L)P_M.$$

The total cost is therefore given by

$$C(L,M)=\left[M-L+\frac{p(1-p^L)}{1-p}\right]^{-1}\cdot\left[\frac{hp}{1-p}\left\{L-\frac{p(1-p^L)}{1-p}\right\}+\frac{hM(M+1)}{2}-\frac{hL(L+1)}{2}+k_0p(1-p^L)+kp^{L+1}\right].$$

Putting $F(L)$ for $$L-\frac{p(1-p^L)}{1-p}$$

and $G(L)$ for

$$\frac{p}{1-p}\left\{L-\frac{p(1-p^L)}{1-p}\right\}-\frac{L(L+1)}{2}+\frac{k_0}{h}p(1-p^L)+\frac{k}{h}p^{L+1},$$

this can be written more compactly as

$$C(L,M)=\frac{h(M^2+M+2G)}{2(M-F)}=\frac{h}{2}\left[M+F+1+\frac{2G+F(F+1)}{M-F}\right].$$

For given L, this is a minimum when

$$0=\frac{\partial C}{\partial M}=\frac{h}{2}-\frac{h}{2}\cdot\frac{2G+F(F+1)}{(M-F)^2},$$

i.e. when $$M=\hat{M}\equiv F+\sqrt{[2G+F(F+1)]}.$$

Substituting this optimum level of M in the equation for C, the least cost for given L is

$$\hat{C}(L)=\frac{h}{2}\{F+\sqrt{[2G+F(F+1)]}+F+1+\sqrt{[2G+F(F+1)]}\}$$

$$=h\{F+\tfrac{1}{2}+\sqrt{[2G+F(F+1)]}\}=h(\hat{M}+\tfrac{1}{2}).$$

The differentiation of $\hat{C}(L)$ with respect to L gives rather an unwieldy expression for optimum L. In practice, however, it is not difficult to discover the optimum level by computing $F(L)$ and $G(L)$, and subsequently deriving $\hat{M}(L)$ and $\hat{C}(L)$, for a range of values of L until the optimum is located to the desired degree of accuracy.

NOTE ON REPRESENTATION OF ISSUES BY POISSON DISTRIBUTION

Experience has shown that the Poisson distribution can often give a good fit to the pattern of issues even when these issues involve variable quantities rather than discrete units. The technique of fitting consists simply of estimating from past data the mean m and variance σ^2 of the quantity issued per unit time, and substituting an "equivalent" Poisson distribution in which issues of constant unit size $U\equiv\sigma^2/m$ occur with average frequency $F\equiv m/U$, so that the mean $UF\equiv m$ and variance $U^2F\equiv\sigma^2$ are the same as those of the original. The use of an equivalent Poisson distribution to represent issues from stock

enables the models discussed in this article to be applied to the determination of least-cost stock levels MU and (in the second model) LU. There is no reason why these levels should be restricted to integral multiples of U, and provided deviation by ± 1 from the calculated optimum values of M and L yields no significant increase in the total cost $C(L, M)$, it seems safe to ignore the bias implied in using a model in which issues are assumed to occur in integral multiples of a hypothetical unit quantity.

NOTE ON REPRESENTATION OF OPPORTUNITIES BY POISSON DISTRIBUTION

In some cases, the intervals between opportunities may be less variable than would have been expected had they originated from a purely random process. In such a case, it may be possible to fit an Erlang distribution, this being equivalent to accepting only every Rth opportunity in a random sequence of opportunities and issues as a "true" opportunity capable of being used for replenishment. The state of the system must then be specified not only by the variable N representing the stock level, but also by a variable $S\,(= 0, 1, \ldots, R-1)$ representing the "phase" or number of "false" opportunities which have elapsed since the last true opportunity. State probabilities P_{NS} can be linked by simple recurrence relations, but the subsequent calculations to find optimum M and L become quite complicated. The model is, however, worth mentioning, as its formulation in terms of a random binary sequence makes it extremely simple to simulate on a digital computer.

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Least-cost Testing Sequence

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If items have to pass each of a series of different tests to be accepted, the cost of testing can be minimized by doing the tests in a certain sequence. A simple rule for finding the least-cost sequence is proved and illustrated.

IN a recent paper[1] Price discussed the arrangement of a series of n non-destructive tests in such a sequence that the cost per item tested would be a minimum. The tests were such that at each stage only those items found to be acceptable were subjected to the next test. It was suggested that the least-cost sequence be found by first calculating the cost per item tested of all $n!$ possible sequences of tests, and then selecting the sequence which gave the least cost. There is, however, a very simple rule for finding the least-cost sequence. The items should be arranged in order of increasing cost per item rejected,

$$\frac{C_1}{R_1} < \frac{C_2}{R_2} < \ldots < \frac{C_n}{R_n},$$

where R is the average rejection rate of a test, and C is the average cost per item tested.

This is similar to the rule given by Gluss[2] for finding the optimum policy for detecting a single fault in a complex system, but we are here concerned with the testing of items which may have a number of faults or none at all.

PROOF OF THE RULE

Suppose we have a series of tests $T_1, T_2, \ldots, T_n$ costing $C_1, C_2, \ldots, C_n$ per item tested, with average acceptance rates $A_1, A_2, \ldots, A_n$ and average rejection rates $R_1, R_2, \ldots, R_n$ (equal to $1-A_1, 1-A_2, \ldots, 1-A_n$). We assume that the C's, A's and R's are unaffected by the order in which we do the tests, and are mutually independent.

If we arrange the tests in a particular sequence $\{T_1, T_2, \ldots, T_n\}$, the sequence will cost $C(T_1, T_2, \ldots, T_n)$ per item tested, and this cost will alter if we alter the sequence.

In order to show that the least-cost sequence is given by the rule, we prove first a lemma and then the main theorem.

Lemma:

If $$\frac{C_j}{R_j} < \frac{C_{j+1}}{R_{j+1}}, \quad C(\ldots, T_j, T_{j+1}, \ldots) < C(\ldots, T_{j+1}, T_j, \ldots).$$

Proof of the Lemma:

Because the A's and C's are independent,

$$C(T_1, T_2, \ldots, T_n) = C_1 + A_1 C_2 + A_1 A_2 C_3 + \ldots + (A_1 A_2 \ldots A_{n-1}) C_n.$$

If we reverse the order of the jth and $j+1$th tests we only change the jth and $j+1$th terms on the right-hand side of the equation. Hence

$$\begin{aligned} &C(\ldots, T_j, T_{j+1}, \ldots) - C(\ldots, T_{j+1}, T_j, \ldots), \\ &\quad = (A_1 A_2 \ldots A_{j-1})(C_j + A_j C_{j+1} - C_{j+1} - A_{j+1} C_j), \\ &\quad = (A_1 A_2 \ldots A_{j-1})(C_j R_{j+1} - C_{j+1} R_j), \\ &\quad < 0, \end{aligned}$$

and the lemma is proved.

Theorem:

If $\dfrac{C_1}{R_1} < \dfrac{C_2}{R_2} < \ldots < \dfrac{C_n}{R_n}$, $\{T_1, T_2, \ldots, T_n\}$ is a least-cost sequence of tests.

Proof:

Consider the transformation of any other sequence of the same tests $\{T_{n_1}, T_{n_2}, \ldots, T_{n_n}\}$ to $\{T_1, T_2, \ldots, T_n\}$ by successively bringing $T_1, T_2, \ldots, T_{n-1}$ to first, second, ..., $(n-1)$th position by repeated interchange of the test being moved with the test preceding it at that stage. By the lemma, the total cost per item tested is reduced at each stage of the transformation. Hence

$$C(T_{n_1}, T_{n_2}, \ldots, T_{n_n}) > C(T_1, T_2, \ldots, T_n),$$

and $\{T_1, T_2, \ldots, T_n\}$ is a least-cost sequence of tests.

EXAMPLE

As an example, we take the example quoted by Price[1]. The tests are listed in Table 1.

TABLE 1

Test	Accept rate	Reject rate	Test cost per item in man-hours	Test cost per item rejected in man-hours	Order of tests for least-cost sequence
Visual	·998	·002	·08	40·0	5
Shock	·95	·05	·584	11·7	4
Electrical	·95	·05	·232	4·6	3
Distortion	·965	·035	·124	3·5	2
Vibration	·35	·65	·64	1·0	1

It is clear that as long as the results and costs of tests do not depend on their position in the sequence, we can easily find the least-cost sequence for a series of tests of almost any length.

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The Economic Distribution of Coal Supplies in the Gas Industry: an Application of the Linear Programming Transport Problem

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The North Western Gas Board bases its allocation of coal supplies from coal mines to gas works, on the solution to a linear programming transportation problem which is calculated at regular intervals on an electronic computer. A complete solution to the problem would require a more general linear programming model but, due to practical limitations with regard to both data and computer, development has been in the relatively simple transportation form. A variety of restrictions have been built into the programme, however, so as to avoid technical production difficulties which might otherwise arise, and in practice the transportation model has proved extremely versatile.

THE PROBLEM OF COAL ALLOCATION

THERE are about 50 production stations within the area covered by the North Western Gas Board, all of which require a continuous supply of coal throughout the year for the purpose of gas-making. About 30 per cent of the coal used comes from Lancashire and 55 per cent from Yorkshire, while the balance of 15 per cent is taken from pits in the Derby and Nottingham areas, North Staffordshire and North Wales. There are about 150 coal mines in these areas from which supplies are obtained in varying quantities from time to time and while the proportion of the total supply taken from each area remains fairly constant, the quantity and quality available at particular pits vary considerably.

Due to the nature of coal mining, the output from a particular mine is limited per unit of time, and the coal mined is not necessarily of constant quality. It follows that even when the total supply greatly exceeds the demand, deliveries to each point of production cannot always be made from the cheapest source. These natural supply limitations have been aggravated in practice, however, by two further factors: first, by the fact that only certain types of coal are suitable for making gas (and coke), and secondly by the general shortage of such qualities of coal since the war.

In general, the types of coal used for gas-making in this area have to be within the size groups known as "doubles" and "singles" (with a decided preference for doubles), although much of the new plant introduced in recent years can operate economically on the smaller varieties of coal, termed "smalls". In addition, the coals must have good coking and carbonizing properties, the most important of these being high reactivity and low sulphur and ash contents.

Up to 1 April, 1959, the shortage of good gas coals (particularly doubles) was such that supplies to the gas industry were virtually rationed. Supplies

were offered on a six-monthly basis by the National Coal Board but the degree of choice was small and was limited largely to the rejection of qualities which were technically unsuitable. Against this background of short supply, there was little scope for increasing productivity and reducing costs by a more economic choice of coals. The application of linear programming was therefore limited to the straightforward transport problem—that of allocating a given supply of coal among the production points within the Board's Area so as to minimize transport costs.

During the year beginning April 1959 the supply of gas coal became more plentiful. Six-monthly allocation of supplies ceased and in general there was a small surplus of coal available relative to the Gas Board's requirements. It then became possible to apply the transportation technique to coal choice and coal allocation simultaneously, considering the cost of the coal at the pithead as well as its transport cost to gas works. The model was adapted accordingly and programming is now being carried out on this basis.

DRAFT PROGRAMME

In order to assemble the necessary data in the form required for the linear programming transport problem—i.e. availabilities, requirements and transport costs—a considerable amount of preparation is necessary. As soon as full information has been obtained about the available supplies, expressed in terms of tons per week, a draft programme of coal deliveries is prepared by the Board's Coal Officer, in consultation with Production Engineers and Works Managers. The preparation of this draft programme is an essential stage in the procedure, as it provides the basis for calculating the correct works' requirements for use in the "transportation" model.

The draft programme is based on the expected average weekly coal requirements at production stations over the period concerned (three months, or six months), including an allowance for building up stocks wherever necessary. Coals are allocated to the works concerned purely from the point of view of satisfying their production and stock requirements over the coming period. To do this, allowance is made for the expected average gas demand, the type of gas-making plant in use (plants have different efficiencies), production policy with regard to the amount of gas to be made relative to the amount of coke produced, and the gas yield likely to be obtained from the type of coal allocated. All factors which have a bearing on the expected coal requirements at each works are taken into account. Coal deliveries are planned, in fact, just as they were before computer programming was introduced with the exception that neither transport costs nor coal (i.e. pit-head) costs need to be considered

DERIVATION OF DATA FOR COMPUTER

At this stage, the draft programme of deliveries includes a number of allocations not considered as transferable, as they are made to meet special requirements and situations, which outweigh the consideration of transport economy

(e.g. the allocation of certain types of coal for making Phimax smokeless fuel). In other cases the most economic allocation is already being made and it is clear that no further changes would be worth while. All non-transferable allocations such as the above are therefore subtracted from their respective works' requirements and are not included in the subsequent programming. The remaining works' requirements, and the amounts of coal supplied to them from the pits concerned, then form the basis from which the data for the linear programming transport problem are derived. The first stage in this procedure may be illustrated by the following programming schedule, representing a small-scale problem with five production stations (the fifth representing the "destination" for amounts not allocated), and five coal mines:

DATA PREPARATION : INITIAL STAGE

TABLE 1

			Tons required at production stations				
	Total		1	2	3	4	5
		11,000	3600	1600	1800	1600	2400
Tons available at coal mines	1	1000	a_{11} 1000	a_{12} —	a_{13} —	a_{14} —	$-c_1$ —
	2	4000	a_{21} 1000	a_{22} 1000	a_{23} 200	a_{24} 1000	$-c_2$ 800
	3	3200	a_{31} 1000	a_{32} 600	a_{33} 1000	a_{34} —	$-c_3$ 600
	4	1800	a_{41} 600	a_{42} —	a_{43} 600	a_{44} 600	$-c_4$ —
	5	1000	a_{51} —	a_{52} —	a_{53} —	a_{54} —	$-c_5$ 1000

a_{ij} = transport cost per ton. c_i = net cost of coal per ton.

Out of 11,000 tons of coal available 8600 tons are allocated to production stations and 2400 tons are unused. The unused quantities are shown as being allocated to "works" No. 5, which has a "requirement" equal to the total amount unused. From mine No. 3, for example, 1000 tons are allocated to works No. 1, 600 tons to works No. 2, and 1000 tons to works No. 3, while 600 tons remain unused.

Transport costs, indicated as a_{ij}, are rail freight and road haulage costs (and, in a few instances, canal transport charges) per ton from mine to works, comprising the established rates of British Railways and road hauliers, plus

handling and unloading charges, etc. Transport costs in column five are zero as this "works" represents supplies not purchased and therefore not delivered. The choice of which coals to reject depends, however, not only on relative transport costs but also on relative prices at the pit-head. These prices may be regarded as negative costs in column five, since coal not purchased contributes a reduction to total cost and, apart from the effect on transport costs, it would be cheapest to reject the dearest coals first (i.e. use the routes with the highest negative cost). This method of dealing with coal prices was used because it is much less laborious and more practical than the alternative procedure of treating these prices as positive, and adding them on to all the appropriate elements in the transport cost matrix (i.e. excluding the last column). The pit-head prices are modified into "net coal costs" to allow for the value of the coke and tar products which are made jointly with gas. These net costs, comprising the pit-head price per ton, less the revenue from the theoretical coke and tar yields, are shown as "c_i" (with negative signs) in column five of the table.

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The programming schedule now appears to satisfy the appropriate definitions for a linear programming transportation problem, in which it is required to minimize the sum of net coal cost and transport cost. Sources and destinations are defined as availabilities and requirements respectively; the sums of the amounts allocated in each row equal the amounts available and the column sums equal the amounts required; finally transport costs are unambiguously defined and amounts transported are necessarily positive. In fact, however, the important proviso that the units transported should be homogeneous is not satisfied. This is because the ultimate requirement at production stations is in terms of amounts of gas, measured in therms (1 therm = 100,000 B.Th.U.), and the types of coal available do not have the same potential value in terms of the therms of gas which can be obtained from them per ton. Consequently, a given allocation in tons can produce a varying quantity of therms, according to the thermal value of the coal used.

It follows that if requirements in therms are to be satisfied, then the whole schedule must be re-formulated with the potential therm as the unit to be transported and with transport and coal costs expressed as costs per therm. To do this, all that is needed is a common standard of measurement in terms of potential gas therms for all available coal supplies. The standard used is the expected yield of gas therms under the operating and plant conditions most commonly prevailing in the North Western Area. (See paper by N. Hare[1], for technical details relating to calculation of gas yields.) All availabilities in Table 1, and all tonnages shown in the body of the table, are now multiplied by the appropriate yields in therms per ton and the columns summed to give the thermal requirements. Each row of costs is put on to a cost per therm basis by dividing by the thermal yield for the coal concerned. The programming schedule now appears as follows:

DATA PREPARATION : FINAL STAGE

TABLE 2

			Therms (000's) required at production stations				
	Total		1	2	3	4	5
		931	310	138	143	135	205
Therms (000's) available at coal mines	1* (95)	95	b_{11} 95	b_{12} —	b_{13} —	b_{14} —	$-c'_1$ —
	2 (90)	360	b_{21} 90	b_{22} 90	b_{23} 18	b_{24} 90	$-c'_2$ 72
	3 (80)	256	b_{31} 80	b_{32} 48	b_{33} 80	b_{34} —	$-c'_3$ 48
	4 (75)	135	b_{41} 45	b_{42} —	b_{43} 45	b_{44} 45	$-c'_4$ —
	5 (85)	85	b_{51} —	b_{52} —	b_{53} —	b_{54} —	$-c'_5$ 85

* Therms per ton shown in brackets.

b_{ij} = transport cost per therm. c'_2 = net coal cost per therm.

The availabilities, requirements and costs are now correctly calculated for the transportation problem, and the data are submitted to the computer in this form, together with restrictions on certain routes, which will be referred to later. The amounts allocated in the draft programme, which are shown in the above example in terms of therms, are not required by the computer as it finds its own initial solution. This pattern of allocations arose, in fact, merely because it was necessary to obtain correct thermal requirements for the works concerned in order to be able to apply the transportation technique.

COMPUTER SOLUTION

The computer solution provides a pattern of allocations which exactly satisfies thermal requirements and minimizes the sum of transport cost and net coal cost. Where surplus supplies are available, as in the example quoted above, the dual solution to the transportation problem is also obtained from the computer. This consists of a set of positive quantities representing the marginal cost of increasing works' requirements by a small amount, and a set of negative quantities which may be regarded as the marginal reductions in total cost obtainable by utilizing further increments of each type of coal, in exchange for others at present allocated, assuming the extra supplies were available. These quantities multiplied by their respective requirements and availabilities form the function to be maximized in the dual transportation problem.[4]

Some intuitive meaning may be attached to the maximization process of the dual problem in this case. With respect to the marginal reductions, if one considers a particular source, the reduction in cost resulting from bringing each successive increment into the solution must gradually become smaller. Successive increments will therefore continue to be brought in, until the source is fully utilized or until no further cost reduction is possible. Hence, sources which are fully utilized generally show some marginal reduction in the dual solution but those only partly utilized or rejected show zero marginal reductions. The marginal costs, on the other hand, must increase as this process advances, since the unused amounts, which would have to be brought in if requirements increased, become progressively dearer.

The list of marginal reductions is of practical use during the periods between runs on the computer. It gives an indication, for example, of the savings to be made by obtaining extra supplies of the coals at present fully utilized in exchange for those partly utilized. The marginal cost reductions are usually referred to as marginal "gains" and where existing supplies are curtailed and alternatives offered it is desirable to obtain the supply with the largest marginal gain which, if possible, should be larger than the marginal gain of the supply given up. The marginal costs and marginal gains together can also be used in conjunction with the transport cost figures to show the minimum price reduction which would make it worth while to bring a rejected supply into the computer solution.

It should be noted, however, that the use of the dual solution as above provides only a rough guide to the best course of action. The marginal amounts concerned (although at least as large as the smallest allocations in the computer solution) may not be large enough to be of practical importance. In addition, in order to obtain the whole of the marginal gain in any particular case, a specific chain of substitutions must be followed and the identification of these substitutions is sometimes difficult in practice. A feasible way of obtaining more exact information about the effect of supply changes, etc., is to extend the computer solution by some manual method while relying on the dual solution as a guide to which changes to test out. It is hoped to put this method into practice soon.

USE OF TRANSPORTATION MODEL : THEORETICAL ASPECTS

The technique of formulating a draft programme in terms of tons, which is subsequently converted to therms and used as the basis for a transportation problem, is valid only under certain assumptions. If all works are operating under standard conditions this approach is obviously correct, but in general there are a variety of reasons for deviating from these conditions at individual works. Some works have different efficiencies as a consequence of the different types of plant used. The two main types of plant are Vertical Retorts and Horizontal Retorts and the gas yield on "Horizontals" is only 84 per cent of that in "Verticals". In other cases, the gas therms produced per ton of coal

may differ from the amounts produced under standard conditions because a larger or smaller quantity of therms are obtained in the form of coke, or other joint products of the gas making process (e.g. benzole). All these variations may be summarized under the term "efficiency", and each works may be assumed to have an efficiency factor expressing the ratio of therms obtained to therms expected under standard conditions.

The programming problem with which we are concerned, and the procedure described in previous paragraphs which leads to the required solution, may be expressed mathematically as follows:

Suppose that in the draft programme, x_{ij} tons are allocated from mine i to works j, and that there are n mines and $m+1$ works (one representing the "destination" for unused amounts). Suppose, also, that actual requirements, in terms of gas produced from the plant at each works, are G_j. Then define thermal values p_i, denoting for each mine the therms obtained per ton under standard conditions, and efficiency factors q_j representing the ratio of therms obtained under actual operating conditions to therms obtained under standard conditions.

It now follows from the draft programme, that since this is prepared so as to provide the gas required at each works, allowing for the current operating conditions,

$$G_j = q_j \sum_{i}^{n} p_i x_{ij}$$

Let the thermal requirements, as calculated from the draft programme in terms of standard operating conditions, be R_j, i.e.

$$R_j = \sum_{i}^{n} p_i x_{ij}$$

Similarly, let the thermal availabilities be

$$A_i = \sum_{j}^{m+1} p_i x_{ij}$$

If the thermal requirements R_j are satisfied, then the gas requirements will also be satisfied, i.e.

$$G_j = q_j R_j$$

Writing y_{ij} for $p_i x_{ij}$, the problem solved by the computer is to find the set of non-negative values of y_{ij} which minimize

$$\sum_{rj}^{n,\,m+1} b_{ij} y_{ij} \quad (\text{where } b_{i,\,m+1} = -c_i'),$$

subject to

$$\sum_{j}^{m+1} y_{ij} = A_i,$$

$$\sum_{i}^{n} y_{ij} = R_j$$

It will be noted that this is a standard transportation problem and that it excludes the efficiency factors q_j.

It is assumed in the above analysis that the efficiency factors do not change during the period in which the programming is carried out (they may do so if production policy changes but this does not normally occur over short periods). Another important assumption is that the factors remain constant whatever the type of coal. If they do not, then the amount of gas actually produced at works j depends on the type of coal allocated:

$$G_j = \sum_{ij} q_{ij} y_{ij}$$

The correct requirements can still be obtained from the draft programme, but the factors q_{ij} can no longer be eliminated so as to derive the transportation model, and general linear programming techniques are therefore required to solve the problem. The available electronic computers, however, could not cope economically with a general linear programming problem with dimensions as large as the transport problems with which we are concerned. Indeed it is the very much simpler form of the transportation model which allows the computer to take advantage of speedy and compact methods of solution so that much larger problems can be solved.[2,3]

In general, it is correct to assume that efficiency factors are constant at each works for all types of coal. Some works do obtain lower gas yields from certain types of coal than from others, but as the use of these coals on the plants concerned has additional technical disadvantages, this difficulty is resolved in practice by applying restrictions to prevent the unsuitable coals from being allocated to these works. The restrictions are applied simply by introducing prohibitively high costs into the cost matrix in respect of the routes concerned. In this way the transportation model is preserved.

OTHER FACTORS AFFECTING THE MODEL

There are two other important factors which have to be considered when programming coal supplies for gas-making, and which, if allowed for fully, would necessitate giving up the transportation model.

First, there is the question of coke production. Since gas and coke are joint products, the changes in coal allocations which arise through adopting the optimum solutions given by the computer may cause undesirable changes in the amounts of coke produced at individual works. This is because the amount of coke normally produced per therm of gas varies among different coals and an excess of coke at some works may cause stocking difficulties (in general only at small works, however, and when only one type of coal is delivered—otherwise, effects of different coals tend to cancel out). In addition, it is necessary to provide certain works with a sufficient quantity of coal of a particular type to supply a specific coke market.

These restrictions in respect of coke production could be allowed for in theory by introducing a limited number of additional linear inequalities into the model, of the form

$$\sum_i \lambda_i y_{ij} \leqslant W_j \quad (\text{or } \geqslant, \text{ as the case may be}),$$

where λ_i is the coke produced per therm using coal from mine i, and W_j is the coke requirement at works j. But in practice, most of the difficulties likely to arise in connection with coke distribution are avoided (and the transportation model retained) by writing prohibitive costs into the matrix. The use of coal with a high coke yield per therm is barred from works likely to have stocking difficulties, and by the technique of splitting up works' requirements the prohibitive costs may be arranged so that particular works receive a minimum allocation of certain types of coal.

The second factor of importance is the need to ensure that a works can always cope with day-to-day variations in gas demand (particularly in peak periods) without deviating from its normal operating conditions. Whether this is possible depends on the speed—known as "throughput"—with which coal can be processed into gas and coke. Some of the smaller-sized coals tend to cause throughput difficulties if used on certain types of plant, with the result that the gas may not be produced quickly enough to meet requirements. With the range of coals normally used, throughput can always be increased, when necessary, to meet the demand but only by operating the plant under relatively costly conditions. This uneconomical working must be avoided wherever possible.

This factor, also, could be allowed for (at least approximately) by introducing additional linear restrictions into the model. The restrictions would be arranged to ensure that the average throughput at a works, according to the coals allocated, did not fall below a specified minimum value. Again, however, the procedure followed in practice is to make use of prohibitive costs in order to retain the transportation model. The coal types likely to cause the severest throughput difficulties are allocated as far as possible to plant where such difficulties do not arise. In addition, some works requirements are split into two, and prohibitive costs used to ensure that these works do not receive more than a certain proportion of small coals.

The following programming schedule (Table 3) illustrates the method of introducing prohibitive costs, which are shown as "M" (for the purposes of this example works No. 5 is no longer assumed to be the "destination" for amounts unused).

It will be seen that the available coal supplies are labelled (s), where applicable, to indicate small-sized coal. In addition, gas-making plant has been classified into Type "A", on which small coal is relatively uneconomic, and Type "B", which takes small coal only. Works No. 1 can receive all types of coal with the exception of the small coal from mine 5, which is technically unsuitable for

INTRODUCTION OF PROHIBITIVE COSTS

TABLE 3

			Therms (000's) required							
		Total	1	2		3	4		5	
		931	A 310	A 69	A 69	B 143	A 35	A 100	B 115	A 90
Therms (000's) available	1	95	b_{11}	b_{12}	b_{12}	M	b_{14}	b_{14}	M	b_{15}
	2	360(s)	b_{21}	M	b_{22}	b_{23}	M	b_{24}	b_{25}	b_{25}
	3	96	b_{31}	b_{32}	b_{32}	M	M	b_{34}	M	b_{35}
	3	160(s)	b_{31}	M	b_{32}	b_{33}	M	b_{34}	b_{35}	b_{35}
	4	135	b_{41}	b_{42}	b_{42}	M	b_{44}	b_{44}	M	b_{45}
	5	85(s)	M	M	M	b_{43}	M	M	b_{45}	M

Type "A" plant. The requirement for works No. 2 has been split into two halves: the first half is not allowed small coals at all, but there are no restrictions on the second half except in the case of coal from mine 5. This ensures that at least half of the requirement is met by the larger-sized coals, so that throughput difficulties are avoided. Works No. 4 must have a certain minimum quantity of coal from mines 1 and/or 4 in order to supply a special quality of coke to a specific market. This, also, is achieved by splitting up the thermal requirement, and all routes in the first part, except those from mines 1 and 4, are prohibited. The requirement for works No. 5 is also divided into two parts, as there are two types of plant in use. Finally, works No. 3 and the first part of works No. 5 receive small coals only.

COMPUTER CAPACITY AND SIZE OF PROBLEMS SOLVED

The first four problems were solved on the Ferranti Pegasus computer but more recent problems were run on the Mercury, which is faster and can deal with larger problems. The capacity limits for transportation problems on these machines are defined, for m destinations and n sources, as follows:

	Pegasus	*Mercury*
No. of routes: $(m.n) \leqslant$	4096	7000 (approx.)
Sources + Destinations: $(m + n) \leqslant$	128	256

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Details of the size of the first six problems solved, including the number of "M" restrictions and the number of available routes, are given in the following Table 4:

TABLE 4

Problem	"Mines" (n)	Works (m)	"M" Routes	Usable Routes
1	48	27	207	1089
2	71	31	805	1396
3	78	27	634	1472
4	93	35	1234	2011
5	102	29	1145	1813
6	87	31	1418	1279

SAVINGS ACHIEVED

The savings to be obtained by adopting the minimum cost solutions provided by the computer may be assessed by comparing the total cost of the computer solution with the cost which would result from basing coal allocation and (where applicable) coal choice on empirical methods.

The first four problems were concerned with coal allocation only. Of these, the first and second were largely experimental and brought to light practical and technical problems in connection with some of the routes used in the minimum cost solutions, which led to the incorporation of restrictions in subsequent programmes. These two problems showed theoretical savings in transport cost of 3 per cent and 2½ per cent, respectively, compared with the empirical pattern of allocation. In the case of problems three and four, both of which showed theoretical savings of about 2½ per cent, the minimum cost solutions were put into practice as far as was possible, and about three-quarters of the theoretical savings were actually realized. A check on actual deliveries over the 11 months (May 1958 to March 1959) covered by problems three and four showed a saving in transport cost of £65,000, due to the use of the computer, compared with the corresponding period of the previous year. The saving, though only small as a proportion of total transport costs, was thus quite substantial and the cost of using the computer (about £300 over this period) was by comparison very small indeed.

Problems five and six minimized the sum of transport costs and net coal costs. It is difficult to estimate, either in theory or in practice, the savings which have been obtained by following the computer solution in respect of coal choice as well as pattern of allocation, because of the considerable change

in supply conditions compared with previous periods. But it seems likely that in the summer period when demand is low and there is maximum scope for coal choice, the rate of saving might well be doubled, while in the peak winter period the rate of saving will at least be as much as was obtained when minimizing transport costs alone.

In considering the savings obtained by computer programming it will be noted that the method of allocation used previously had already achieved a high level of economy, and that the proportional amount of saving due to the use of the more scientific method is relatively small. This is attributable partly to the fact that a great deal was already being done, within the limits set by the empirical methods available, to seek out the most economic patterns of allocation. It also illustrates, however, that when a problem of the transport type occurs in practice as a permanent feature of a business, the so-called "rule of thumb" method can go a long way towards the optimum solution. This is particularly true if the general pattern of requirements and availabilities remains broadly the same from one period to the next. In such a situation, once a reasonable pattern has been established, any changes which are suggested due to individual preferences and other circumstances are likely to be accepted only if they lead to a reduction in transport costs. Thus, in favourable circumstances there would always be a tendency to approach the optimum solution. If radical changes in availabilities or requirements are the feature of a transport problem, and provided the new pattern then remains constant for a known period, it would appear that the proportionate savings obtained from the use of linear programming are likely to be larger.

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Psychoanalytic Contributions to an Operational Research Study of Marketing*

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The findings and experiences reported here are not unique. Indeed, following presentation of this paper, a member of the audience apprised us of a somewhat similar study of a large American meatpacking firm by a psychiatrist and a team of economists. The psychiatrist and the economists separately studied the Board of Directors and attempted to predict the decisions which the Board would make on problems before it. The psychiatrist consistently predicted the Board's decision more often than the three economists, knowing only the personality structure of the individual members and their positions on each problem, but not knowing the economics or profitability of the alternative courses of action. These latter were the bases of the economists' predictions.

In another case, MELVIN THORNER, M.D., Sc.D., also a psychiatrist, studied the interpersonal dynamics of a group of engineers in a metallurgical company. He was able to predict decisions of the research department also on the basis of his understanding of the personality of the individuals responsible for the decisions, and showed that the decisions often were independent of the technical merits of the problems, and which subsequent technical results verified.

In a separate study reported last week at the meeting of The Institute of Management Sciences in Paris, we presented results which indicate that managerial decisions in general are "projective" in the psychological sense. The value of these findings is to help clarify the ways in which unconscious needs and attitudes manifest themselves in emotions and in the whole life process, including in the development of the individual's life style, whether in doing operational research, in managing, or in any other area of human endeavour. If the individual understands and reduces the distraction induced by these needs and processes, he becomes able to direct more of his energies, intelligence, and resources to more productive and profitable channels. Every life experience, including doing operational research, offers opportunity for gaining this increased understanding.

Much work has been done recently on discovering the consumer's conscious and unconscious motivations for buying particular products. This information helps the seller to manipulate the buyer. Its orientation thus is the opposite of the orientation of the several studies reported here. These are rather concerned with helping the individual to understand himself in the full depth of his rich personality and to permit himself thereby not to be manipulated either intentionally or fortuitously, as by his seating arrangement, chair style, co-workers' attitudes, or his own obsolete needs and attitudes. Thus, these studies are directed toward results which help him to enrich the scope and content of his life experience to his full potential.

* Presented at joint meeting of the Operational Research Society of the United Kingdom and The Institute of Management Sciences, London, 14 September, 1959.

INTRODUCTION

THIS paper presents the results of our experiences in participating in the development of a large-scale data processing and management decision making system for a company in a consumers' goods industry. We believe it illustrates the relevance and potentiality of psychoanalytic concepts and techniques for improving both the process of executive decision making *per se* and the study of such decisions by operational researchers. We analyse examples in which unconscious motivations and needs appear to affect both executive decisions and the performance of operational research studies.

The paper is in two parts: the first contains a highly condensed psychological model of human behaviour, with some particular reference to those branches of human behaviour called "executive decision making" and "operational research". In this part we endeavour to construct a model in which behaviour is modally a function of selected conscious and unconscious needs, motivations and drives.

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The second part addresses itself to several experiences in which we have used the concepts of the model and related techniques and insights to influence decisions and operational research methods. We do not assert these experiences "prove" the realism or relevance of the model. These we feel have been proven elsewhere to reasonable standards of clinical and scientific objectivity and reliability. Thus, the reader may take the second part as an interesting narrative supporting either his model or ours. We used these experiences to convey the concepts of our model; we have found that first-hand participation is virtually the only way of communicating them. They are cited here to give the reader at least partial insight into the potential of these concepts.

DETERMINANTS OF DECISION

The first idea we want to present and illustrate is that executive decisions in business are affected both by conscious and unconscious mental processes and concepts. These include familiar logical methods as well as individual emotions, group attitudes, and the history and institutional mores of the organization as these are contained in ideas, feelings and attitudes inculcated by the organizations' founders, participants, and leaders. These processes and concepts may be conscious, i.e. in direct awareness, or unconscious, i.e. not in direct awareness and experienced only through emotional arousal.

The second idea that we wish to present is that even in doing "objective" operational research on executive problems, the operational research scientist's unconscious mental processes often affect the solution of the problem. They may affect his method of study, what he accepts in the study, how he collects and analyses his facts, his point of view, and his perspective. In addition, he often justifies his unconscious attitudes, feelings, and objectives by the familiar, but the unconscious process of "rationalization". Thus, the operational research scientist joins with the human race in that he does his work according to his

unique point of view and perspective. This perspective is part of his whole "life style", which he has created, largely unconsciously, and which serves to stabilize and support his personality; in the same manner, the life styles and prototypal roles of all other humans serve to stabilize and support them also. The individual life style of each operational research scientist manifests itself in his work in many ways. For example, that the person has become an operational research scientist is itself a manifestation of a particular, unconscious need structure. His doing operational research is a continuing rationalization and justification of the particular reality which he has created in part to justify his set of unconscious needs, attitudes and drives.

The third idea here is that psychoanalysts participating in an operational research study as members of the human race are not exempt from the ideas described above. Indeed, the experiences to be described later in this paper illustrate how the psychological interpreters also have to be constantly aware of their own needs and motivations and their own rationalizations as part of the feedback to the operational researchers if they are to help them to maintain realistically greater objectivity in the operational research work.

Our experience indicates a scientist or operational researcher tends to do his work the better if he has a trans-cultural perspective, considerably broader than that typically held by the so-called organization man. The effect of the unconscious "trap" on one's work can be avoided best if a certain distance is taken from one's own needs and motivations and those of the organization in which he is participating. Thus, through "interpretation" of his needs as he manifests them in his life the operational research scientist can become aware of his unique psychic need structure, and does not attempt to impose it upon, or limit his work on, the problem which he is studying for the organization. Similarly, he is able to avoid permitting any other person to impose that person's needs and motivations upon him, except where these are conducive to objectivity and effectiveness in his work.

The ideas we are presenting here can be taught, acquired and used. They involve both conscious and unconscious learning and reinforcement to alter both conscious scientific work methods, as well as unconscious needs and motivations in that work. In this regard, we emphasize that our orientation and focus are not therapeutic. Although psychoanalysis ordinarily is concerned with the alleviation of emotional conflict, the object here is different. The concepts and methods of psychoanalysis are useful in at least the following ways: (1) as a theory of personality, (2) as a psychology, (3) as a research tool, and (4) as a form of therapy. Therapy is only those instances in which the goal is a change in the character structure and personality of the individual in order to alleviate unconscious (emotional) conflict, to enhance his personal potential, and to facilitate the actualization of his potential.

Our goal differs. We seek only to help the individual (or the organization) to become aware of his unconscious needs and motivations and the resulting

paratactic distortions which unconsciously introduce themselves into his conscious processes, his interpersonal reality, and his dynamic interaction within groups and organizations. Thus, he is able to cope with both the scientific and the organizational problems with greater objectivity and realism. He then can include personality structures he encounters and the histories of their development in his operational research studies as part of the data of the problem. He is not necessarily the decision maker, but he can become a counsellor to the decision maker on broader issues. In this sense the operational researcher is an active participant-observer, but with a new degree of objectivity and realism in his contribution to the problem analysis and decision making process.

The psychoanalytic perspective which we are using in this study combines both interpersonal and intrapsychic schools, i.e., the Freudian or the intrapsychic school, and the Horney-Sullivan or interpersonal school. We endeavour to focus on the whole living experience as a dynamic process in time and space, involving interpersonal relationships, institutional mores, intrapsychic phenomena, and geo-cultural mores. Of course, ours is but one psychological model of reality which can be constructed. Every scientist can and does construct whatever models of reality that may be useful to his work. We find this particular model useful because in a relatively simplified way it isolates key factors or variables of interest and it includes well-developed methods and techniques for manipulating the variables of interest. Of course, there may be many circumstances in which others may find different models useful to them, and we make no assertion that such other models have less validity. In this sense, we are not speaking as if our model were the only and absolute model, that it alone is realistic. Rather, we only suggest that the ideas which we present here can become new tools of the operational research scientist to increase the power, scope, and utility of the contribution which he can make to society, to his employer and to himself.

In order to interpret our observations in the following operational research study we need summarize some further psychological ideas. Dr. Karen Horney has described the "basic neurosis" (or perhaps origin of the basic neurosis) as the infant's need to be accepted, approved, loved by his parents, parent surrogates, siblings, etc.* The need is seldom wholly satisfied and continues only partially fulfilled into adulthood as a need, instead of as only a mature want. Where the need requires immediate satisfaction whenever aroused, the want can be delayed. The unsatisfied need induces emotional and always unrealistic response; the unsatisfied want induces realistic, goal-seeking behaviour devoid of debilitating emotion.

Whether a particular person's behaviour is need or want induced is apparent least of all to him. Through unconscious processes such as rationalization, projection, etc., he adopts attitudes and behaviour which accord with his

* Other needs and wants have been identified and are relevant, but are of secondary concern here.

need, and which then he justifies in order to stabilize and support his personality in the roles. This behaviour includes, for example, the life style described earlier, including the decision to be an operational research scientist, a manager, or a Member of Parliament. To the extent that a need is operative, the behaviour and life style serve to maintain personality homeostasis, and induce any of a whole spectrum of primary, secondary, and/or tertiary defences. These range from direct emotional response and stabilizing modes of behaviour, to various reality distortions and finally to detachment from reality.

There are three "pure" modes of behaving, or experiencing oneself, i.e. personality prototypes; the aggressive type, the compliant type, and the withdrawn type. Any person tends to exhibit some combination of these in accord usually with genetically or infantally induced factors. Associated with each type are both direct and defensive tendencies. For example, the aggressive person may "appear" compliant on the surface, but this is only a reaction formation he uses to control his basic tendencies. In any case, the cueing off of the individual's mode of behaviour is his effort unconsciously to satisfy the basic need: to be accepted, approved, loved. This tends to be true of the behaviour of Mr. Milquetoast, of the hard-boiled Marine sergeant, of the frustrated housewife, of the hard-driving executive, of the esoteric scientist or of the fast-talking sociable salesman. Their unconscious goal is the same; their behavioural or self-expressive paths differ.

Status, position, location, dress, title, role relationship all represent, consciously or unconsciously, some form of temporary satisfaction of the basic need. To the extent that the need is operative and is satisfied by those acceptance symbols, the individual is temporarily relieved of having to spend at least some (or all) of his energy, intelligence, and attention to supporting and justifying himself. Thus, he can direct his resources, at least temporarily, to the realistic problems at hand.

In the following we describe how these need structures affected operational research methods and executive decisions.

DESIGNING A MARKETING SYSTEM

This study was made in co-operation with the vice-president of a large American manufacturer, in the marketing of consumer products. His market includes the whole of the United States. His product line is diversified, and the products are relatively durable after manufacture. They are used almost continuously by the consumer. The marketing manager had organized an operational research team to study and to design systematically the whole marketing structure for more effective operation. This included the decision guides and operating procedures for the salesmen, sales supervisors and regional sales managers, criteria and guides for advertising and sales promotion, the information and data processing system for the collection, analysis and presentation of marketing information, the inventory and flow of the product, and

the construction of macro and micro economic models of the marketing regions to analyse the consumption and use of his product line. The end objective of the study was an integrally designed marketing system, including advanced training of sales and market personnel in the new concepts and methods. The training was both to increase their sales abilities and to do so in a way consistent with the broad, overall objectives of the company.

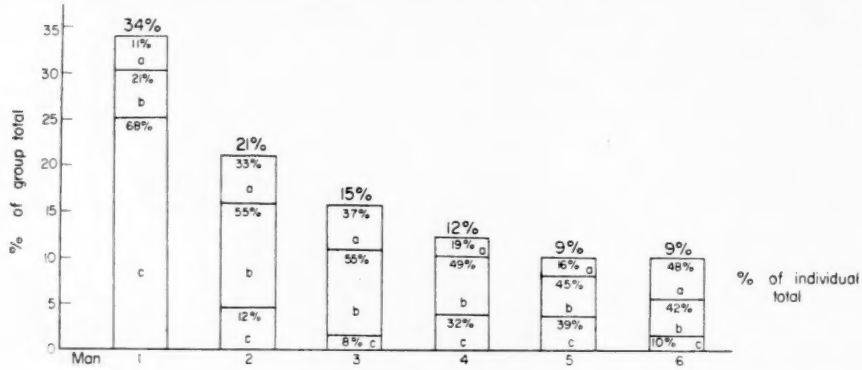

FIG. 1

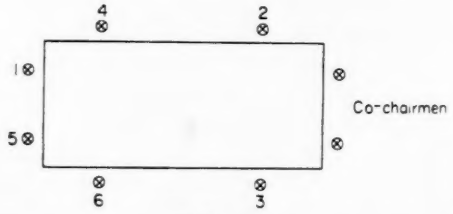

FIG. 2

In the operational research study, the psychic needs as they operate through the personality types of the participants were observed to affect the work and decisions in the following illustrative ways. During a typical discussion, we served as alternating chairmen of the group. Our role was to review the technical work, to comment and make suggestions, and to aid in the development of work climate. While not serving as active chairman, each of us alternated in studying and recording the behaviour and attitudes of the group. In one morning session, the following distribution of relative frequency of contribution to discussion was observed. Each contribution was categorized into (a) a new idea or alternative to be pursued, (b) a reinforcement of an idea under discussion, (c) a criticism or rejection of an idea under discussion. A fourth category not recorded quantitatively was (d) initiation of a group decision to act upon an idea. This will be useful in subsequent studies. The distribution of contributions is shown in Figure 1.

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During this discussion, man No. 1 obviously dominated, and tended to cause his end of the table to be a focus of attention. It is a strong tendency for a person who tends to disagree also to provide relatively more distance (both physically and emotionally) between himself and the others. Both are evidences of the child who disobeys because he "knew" (unconscious anticipation due to prior conditioning) he would be rejected anyway, and who detaches himself from the group with hostility before the group could do so. If the group moved away from him, it would confirm and arouse his unconscious fears of detachment. He fears the arousal by others so he does it first. Men Nos. 4, 5 and 6 tended to move to their lower positions at the table as self-effacing tendencies, reminiscent of the self evaluations fostered on them by their parents and of the methods they adopted for handling their appraisals of self worth. Nos. 2 and 3 moved to the head, in accordance with their self-expressive tendencies which were apparently not unconsciously inhibited. The professional training of each man was:

Man No. 1. Ph.D. in Economics.
„ „ 2. M.S. in Engineering.
„ „ 3. M.B.A. (Business Administration).
„ „ 4. Ph.D. in Statistics.
„ „ 5. Ph.D. in Psychology and Statistics.
„ „ 6. Ph.D. in Applied Mathematics and Numerical Analysis.

These personalities and their arrangement as in Figure 2 led to distribution of contributions as in Figure 1. They reached a group conclusion in which opposing ideas were negated by man No. 1. The group acquiesced in his proposal for routine microeconomic and inventory control computations in each of the sales districts.

In preparation for the afternoon session, we suggested the seating be randomized. We then moved the participants to positions shown in Figure 3.

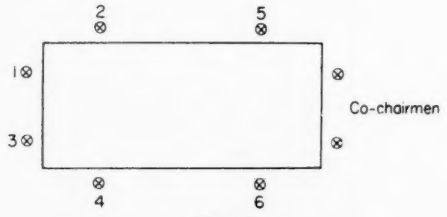

FIG. 3

In addition, Nos. 5 and 6 were given arm-chairs similar to ours, while the others continued to use straight chairs. By this move, the lesser contributors gained relative status at the table and some halo effect associated with their chairs and their new positions near the chairmen. No. 1 was surrounded by

more assertive persons. The resulting distribution of contributions during the afternoon is shown in Figure 4.

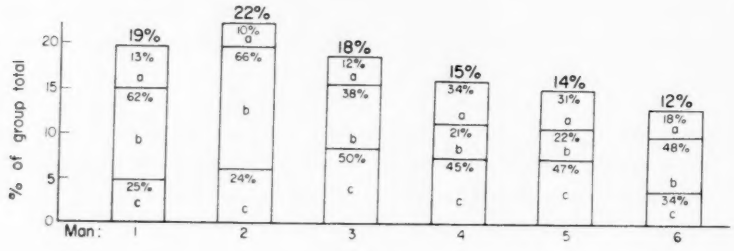

FIG. 4

The decision which had been made in the morning was reversed. The interpersonal dynamics of its reversal were that man No. 1 was constrained to be more compliant in proximity to Nos. 2 and 3. He could not express his hostility nor detach himself as freely. While the burden of the counter argument was not carried by Nos. 2 and 3, the restraint imposed by their proximity to No. 1, together with the increased status of No. 6, made it possible for him to express his views more freely and fully. From a mathematical and computational point of view, No. 6 demonstrated that the economic analysis and inventory models could not be handled, even though they were economically desirable. Even their large-scale computer did not have the capacity. While the logical persuasiveness of his views eventually would have carried when the proposal was attempted, the purpose of the discussion was to bring best judgement to bear and to agree on a course of action. However, as is often the case, No. 1 might have continued with his proposal long after experience had cast doubt on it, in an effort to justify his earlier position.

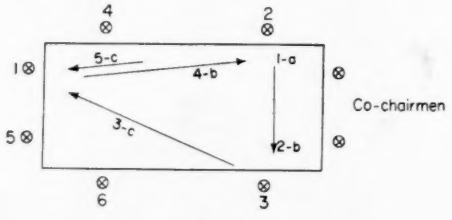

FIG. 5

It is worth noting that the overt behaviour and attitude of the group was friendly and discoursive. Although man No. 1 had evidenced a high percentage of negative responses, he maintained a scholarly appearance and demeanour. However, this made his highly composed criticisms and evaluations of proposals more effective, and resulted in restraining others from speaking as fully as

they might otherwise. His criticisms raised the others' unconscious fears of rejection and cued off their feelings of low self-worth. These fears and feelings induced the individuals so affected unconsciously to rationalize their arguments so as to avoid further arousal of the fears.

By rearranging the group, the status symbols of the new arrangement counterbalanced to some extent the differences among the participants in the strength of their self-expressive tendencies. In effect, we apparently had manipulated the group by selection of seating and chair arrangements and, thereby, had influenced the decision which it made.* After these technical sessions, we began introducing our observations on their group behaviour, including the dynamics and patterns of their interpersonal relations. For example, a typical pattern of these responses was:

In this exchange of ideas, man No. 2 originated a suggestion designated (1–a) where "1" designates the ordinal position of the contribution in this sequence, i.e. No. 1 and "a" designates from Figure 2 that it was an original suggestion, man No. 3 made the second contribution of type "b", etc. It "died" with No. 2's second negative response.

At the same time, we began to introduce them to the unconscious mechanisms whereby their needs affected their conscious processes and attitudes, as through projection, rationalization, introjection, and transference. Next, we gave them cues to observe in interpreting one's own and others' emotional states and in tracing the origins of those emotions. Finally, we began to expose them to their individual patterns of behaviour observed in the conferences, and gave them experience in recognizing and interpreting those patterns. In due time, they developed considerable skill in these analyses which enabled them to increase the objectivity of their individual and collective work. Indeed, they increased their skill in relating to others in the organization and in maintaining a climate which permitted more effective communication with those others. No longer, for example, did scepticism, hostility, or avoidance by others who were not familiar with their operational research work arouse their own emotional defences.

In the beginning, the leader of the group had adopted the job title "Co-ordinator of Operational Research", in preference to "manager". This was part of his defence mechanism of avoiding spontaneous self-expression as in a positive leadership role and of unconsciously seeking approval of his personnel by avoiding leadership decisions and behaving in a compliant manner.

* We do not discount the possibility that other factors also might have affected the reversal. But the result is consistent with our experience and expectations in similar situations. See, for example, HOLT and WINICK, "Seating Position as a Psychological Synecdoche in Group Analysis: A Study in Sessology": "Psychiatry", *Journal of American Psychiatric Association.* See also BERNARD COHEN in an unpublished paper, who found that "in a rectangular group the people at the two ends would be 'higher' talkers than those in the middle". Also, STEINZOR found "a significant relationship between the distance of one discussion participant from the other to follow the other with a verbal comment": *Journal of Abnormal Social Psychology*, **45**, 1950, pp. 552–5.

Fortunately, his behaviour changed as his staff's also changed, so that he began to be able to exercise his proper managerial authority as the staff began to be able to accept that authority without transferring obsolete attitudes on to authority figures. He also increased his capacity for creative self-expression of his own superior abilities.

During the discussions we, as humans also, had emotional responses and were actively concerned with assuring that we maintained our own objectivity and performed our required prototypal roles. Firstly, as "outside authorities" we were immediately the recipients of the unconscious attitudes which the group members unconsciously felt toward authority figures and symbols. These attitudes were expressed toward us as symbols, not at us as humans. Thus, some members expressed overt or subtle hostility; some exhibited ingratiating compliance; others tended to withdraw. Also, there were reversals: as a hostile expression would lead to guilt feelings, a member might tend toward placating compliance, which soon would return to hostility. Importantly, these attitudes were expressed before we had had an opportunity to cue them off by our actions; they were based upon the bearer's unconscious attitudes toward authority.

As part of our task, it was necessary to maintain a co-operative, objective approach and not allow any of these attitudes to influence our evaluations or ideas. Thus, we too had to examine automatically any of our emotionality, to place each instance of emotion in perspective, and to dissociate the emotion from the individual or the issue being discussed.

In the further discussions which ensued, there were varied projections toward us. For example, because we were "authorities", unrealistically complex questions were given us, a subtle form of hostility. Or, questions would be asked which were really their own answers and were intended to cause us to take sides, a dependence manifestation. Our role was to help elicit objective information. Thus, in each instance, we too had to re-examine our impulses to assure that momentary satisfactions, for example, "playing the role of consultant or authority", did not detract from the objective goals which had been established for us.

In the dynamics of the discussions, we too had impulses to join one side or the other, largely based upon emotional responses, either affirmative or negative, to the individuals and their personalities. Such, however, would have reduced our effectiveness as objective counsellors on the programme and as objective interpreters of the group's behavioural pattern. For example, we tended to have a positive effect toward man No. 6 (i.e. we liked him), but if we had expressed it in preference to certain others, it would have disrupted the group's spontaneous dynamics and eliminated the possibility of our presenting them with an objective picture of those dynamics. That is, if we had returned a person's projected hostility, we thereby would have confirmed his unconscious presupposition that we were hostile. Or if we had reacted

supportingly to the dependence, we would have confirmed that we were paternal figures on whom to depend (and later to rebel against). In addition, we felt that indirect measures, such as rearranging chairs, etc., might make it possible to aid man No. 6 to make his point without our overt psychological support. (Of course, we expressed our technical views as appropriate, but avoided letting them become issues in the behavioural dynamics.)

There were many other topics discussed and evaluated beyond the micro-economic computations. These included inventory control, information display for decisions, development and computation of criteria for operating decisions, simulation of operating policies, etc. These were covered in weekly meetings over a period of several months. The group's dynamics during this period shifted several times, but always moved toward fuller self-expression by the members and more objective evaluation by the group of the issues before it. They acquired the ability each to observe the dynamics of the group and his role in them, and to be able to help the group to dissociate unconscious needs from the objective issues at hand.

Letters to the Editors

RIVETT's "Survey of Operational Research in British Industry"[1] raises two points.

Firstly, can we call operational research those studies which are "sophisticated crystal gazing"? Operational research has three stages of research; (a) preliminary investigations of a problem leading to the isolation of the area where the attack is to be made; (b) a detailed study of the area leading to a hypothesis or tentative conclusions; and (c) the study on the performance of the conclusions arrived at when put into practice. The three stages cannot be divided and no operational research study could be complete without the latter.

What is the cause of this lack of balance towards which Rivett, Duckworth[2] and others have pointed out? (a) Lack of publication of those papers and reports where models were tried in practice and yielded results, arising out of the practices in the Industry or (b), attitude of scientists who limit their study to neat solutions and leave out the involvement in practice, or (c) conceptual limitation of the subject? It may be due to all three causes, or it may be due to more than that.

It may, however, be stressed here that the early definitions of operational research, which limited its role to creating a quantitative basis for decision making, may have a greater share than may appear at first sight. The intellectual outlook and the climate of opinion this definition created laid the rails on which further development has been proceeding. The compartmentalization of operational research to the use of a few specialized techniques has been a necessary consequence of this development, rather than a lack of balance.

A contribution in *Operat. Res. Quart.*[3] in 1952 and a letter by Norman Hitchman in J.O.R.S.A. in 1953[4] pointed out this deficiency. The results of a different approach followed in a laboratory in India[5] and specifically in the field of utilization of cotton seed and its by-products would give another interesting picture.[6] The studies are continuing and are by no means complete at this stage.

The second question which it raised is organizational. What is the role of professional societies, can they assume the role of direction? Apart from pooling of experience, the promotion of a synoptic view of a subject and the consequent effort in giving a suitable direction to research should also be the proper role of Society meetings, but they seem to shy away under the pressure

of technical papers to be discussed. This point and the one raised by OBERBECK[7] in America do need some serious thinking.

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[4] J.O.R.S.A. **1** (1953), No. 4: "What is the Mission of Operations Research?"
[5] *Operat. Res. Quart.* **10** (1959), 118: *News and Notes, Regional Research Laboratory, Hyderabad, Ann. Rep.* 1957–58.
[6] Proceed. of the Symp. on Cotton Seed and its By-Products: Operational Research in Planning of Research, p. 339. Indian Central Oil Seeds Committee, Hyderabad (India), 1959.
[7] *Operational Research* (J.O.R.S.A.) **7** (1959), 813: "Some Information on the Organisation of Scientific Societies", T. E. OBERBECK.

Ballade of Irrational Limitation

"They shall not pass!", the Verdun heroes cried:
Still that heroic cry is heard to-day;
The I.F.O.R.S., with foolish pride,
Keeps lesser mortals stubbornly at bay.
"None but the true practitioner", they say,
"May come to Aix and meet our splendid crew".
Shall Britain's members answer yea or nay
For limitation to the favoured few?

Witches and wizards keep the world outside
As, with weird rites, to pagan gods they pray—
Must we, like necromancers, slink and hide
With harmless Queues and worthless* Games to play?
Shall we deny to those of humbler clay
The glorious chance of meeting me and you
(Especially if they're prepared to pay)
By limitation to the favoured few?

Are we afraid they'll come from far and wide,
From Nether Wallop, Cork and Stornoway,
To learn how maths. and logic are applied—
Only to find how oft they gang agley?
And see austere professors going gay
Toasting St. Maximin in local brew?
They'll know the reason, seeing us at play,
For limitation to a favoured few.

L'envoi

Prince, application here from Mr. K.
And, Heavens! one signed "H. Macmillan", too—
"Tell them, Miss H., that they must stay away:
Strict limitation to the favoured few".

* i.e. zero-sum.

A. BATTERSBY

The Secretary of I.F.O.R.S. regrets that he cannot reply in the appropriate manner; he gave up writing doggerel many years ago! The integrated value of a conference as a function of size has been shown to go through a maximum at a size somewhat (probably 50 per cent) greater than that corresponding to the maximum value of the discussion (see theory of graphs). The size planned for the Aix-en-Provence Conference is more than twice as great.

The Secretary, I.F.O.R.S.

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Abstracts and Reviews

Communication with the Patient.

A. MEARES.

The Lancet, **1**, 7126, *pp.* 663–667 (26 March, 1960).

There is a multi-channel system of communication between patient and physician. These different channels are formed by literal verbal, extraverbal, and non-verbal modes of communication. The physician receives different messages through different channels at the same time. His attention to verbal communication should not lead him to disregard information available through the other channels. Some ideas can be effectively communicated only by non-verbal means. To understand his patients the physician should be competent in the language of non-verbal communication.

J. M.

Control of Stocks by Simple Rules.

R. H. COLLCUTT.

Engineering, **189**, 4895, *p.* 232 (February, 1960).

A general description is given of a case study from the paper by Collcutt, Banbury, Massey and Ward, *Operat. Res. Quart.* **10** (2), 81–95 (1959). Simulation of different re-ordering policies led to the formulation of a set of practical rules by which the mean and variation of stock level could be reduced. The solution is not claimed to optimal, but to be much better than had been used previously.

C. W.

Process Simulation—BP's New Computer.

The Petroleum Times, **64**, 1635, *p.* 257 (8 April, 1960).

A brief description of a new analogue computer (CERES) installed by the British Petroleum Co. Ltd. to be used for process simulation. Its main uses are to study ways of improving existing processes as, for example, problems of instrumentation and control and in the early design stages of future projects. Some details of the design are given.

H. R. W. W.

Heating and Ventilating Research Laboratory.

The Engineer, **209**, 5438, *p.* 652 (15 April, 1960).

Since its inauguration on 29 December, 1955, the Heating and Ventilating Research Council has, as a temporary measure, rented laboratory space in the

laboratories of the British Coal Utilization Research Association at Leatherhead. Attention is being paid to operational research to effect economies in installation costs by standardization and prefabrication of parts and procedure. An example of such work is the queueing simulator, which has been designed to show at what point an increase in utilization of a screwing machine may become uneconomic owing to queueing. Balls of various sizes represent a range of similar jobs, each of which requires a specific period of time on a screwing machine as represented by a slope down which the balls run in a known time. This is proportional to the time spent at the machine. The balls are introduced into the system at random intervals, to represent the arrival at the machine, a rotating disc providing the sequence, and after arrival the balls pass to the queueing area and are held there until the machine is free. By using the simulator the average queueing time can be found for different utilization rations and the scale time is such that a day's work can be simulated in about forty minutes.

M. B. C.

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Some Remarks on the Game "DAMA" which can be played on a Digital Computer.

N. V. FINDLER.

The Computer Journal, **3**, 1, *pp.* 40–44 (April, 1960).

The formulation of some evaluation functions for use in the game "dama" (a form of checkers), and its programming for Silliac, is described. A learning process in which the computer plays against itself, using these functions, is suggested.

B. H. M.

Selling Safety: an Experiment on the Effect of specially designed Safety Posters.

Safety, No. 8, *pp.* 21–24 (February, 1960).

Three specially designed posters urging slingers to hook back chain slings when not in use were prominently displayed in 22 shops of 7 steelworks. Frequent counts were taken of the number of slings properly replaced for five weeks before and for six weeks after the posters went up. During the six weeks of poster display, the frequency of the safe practice enjoined by the poster rose in 17 out of the 22 shops. The increase was greater in low roofed shops where omission of the practice spells greater danger. The evidence further favoured the serial display of the set of posters at fortnightly intervals as against the simultaneous display. A follow-up eight weeks after the conclusion of the main experiment showed further increases.

S. L.

Putting Two and Two Together.

DENIS THOMAS.

Safety, No. 8, *pp.* 31–34 (February, 1960).

The drawbacks of the so-called "league table" system for plotting and presenting accident frequency rates used in many steelworks and elsewhere, and of an

alternative method of simply plotting frequency rates at monthly intervals are compared with a quality control type of chart. In contrast to the first two methods, the accident control chart which contains warning and action limits shows up overall trends towards improvement and deterioration in the accident position whilst discounting purely chance fluctuations. Description of the method is accompanied by an illustrative example.

S. L.

On comparing Two Observed Frequency Counts.

M. KUPPERMAN.

Applied Statistics, **9**, 1, *pp.* 37–42 (March, 1960).

Two random samples of size N_1 and N_2 are drawn from c-valued populations. Let the observed frequencies for the first sample be $f_1, f_2, \ldots, f_c$ and those for the second $g_1, g_2, \ldots, g_c$. To test the hypothesis that the populations are the same we may use either a simplified expression for χ^2, viz.

$$\chi^2 = \frac{(N_1+N_2)^2}{N_1 N_2}\left(\sum_1^c \frac{f_i^2}{f_i+g_i} - \frac{N_1^2}{N_1+N_2}\right)$$

or

$$2\hat{I} = 2\left\{\sum_1^c f_i \log_e f_i + \sum_1^c g_i \log_e g_i + (N_1+N_2)\log(N_1+N_2) - \sum_1^c (f_i+g_i)\log(f_i+g_i) - N_1 \log N_1 - N_2 \log N_2\right\}$$

It is claimed that the latter is easier to compute now that tables of $2n \log_e n$ are available. It is distributed approximately as χ^2 with $(c-1)$ degrees of freedom.

D. J. B.

An Application of Linear Programming to Agricultural Economics.

SHIRLEY E. VINCENT.

Applied Statistics, **9**, 1, *pp.* 28–36 (March, 1960).

The linear programming technique is used to find the best organization of a dairy enterprise. Net profit is expressed as a linear function of 93 variables and this must be maximized subject to 27 restrictions (mainly dietary requirements). The simplex method is impracticable without an electronic computer, so a modification—the Matrix Inversion method—is used. The success of this method depends upon being able to predict fairly accurately which basic variables will appear in the optimum solution. This is possible in the application considered. The solution is used to find optimum programmes for conditions on other (hypothetical) farms.

D. J. B.

Planning and Justifying Capital Expenditures.

Edited by ARTHUR LESSER, Jr., STEVENS INSTITUTE OF TECHNOLOGY.

Papers and discussions of the 1959 Summer Symposium of the Engineering Economy Division—American Society for Engineering Education and the Engineering Economy Research Committee of the American Institute of Industrial Engineers.

Nine main papers, six discussion papers, a summary of the papers by the editor and a booklet to be found in an envelope stuck to the back cover are the varied ingredients of this American dish which was first served in the summer of 1959. As is to be expected, the style, treatment and content of each paper differ as widely as the background of the authors, but together they give a helpful and sometimes stimulating approach to the fascinating subject of planning and justifying capital expenditures.

All the papers are concerned with planning and justifying capital expenditures from an economic viewpoint, and one is impressed throughout with the difficulty of making correct decisions in the face of uncertainty. The businessman might applaud the suggestions for simplifying such decisions by the use of predetermined formulae, but is unlikely to accept and apply such formulae uncritically and unrealistically. The formulae themselves are interesting and an understanding of their bases may lead to more scientific and successful decision taking.

The well-known authority on capital expenditure, GEORGE TERBORGH, contributes the first paper (and the booklet in the back cover) on project justification. He introduces a revised and improved method of arranging capital projects in order of profitability. To those familiar with the formula which first appeared in 1949 in "Dynamic Equipment Policy", the new approach has many similar aspects, plus the advantages that jargon has been removed and the formula can be applied to new as well as replacement investment. Of as much interest as the formula itself is the sane down-to-earth approach towards it.

The next paper on capital budgeting in the General Electric Company (of America) outlines the organization and procedures of the budgeting process and highlights the need to look five, ten and even twenty years ahead where large capital programmes are envisaged.

Two papers follow on checking estimates used in capital budgeting. Most businessmen acknowledge the need for this as an aid to future planning, but it is unusual to find it done in practice. One paper attempts to remove the sting from post audits (as they are called) by using conventional data. The discussion paper was not impressed with the validity of doing so in relation to one particular project out of many.

Capital budgeting has always placed emphasis on fixed asset expenditure but rarely on working capital. Additional working-capital requirements may turn a profitable investment into an unprofitable investment and should not be neglected. The estimation of working-capital requirements for long-range planning is discussed by two members of the Atlas Powder Company.

Of lesser interest to businessmen and others in this country are two articles on the effects of taxes on investment decisions. The taxes considered are those of the U.S.A. and in view of this the articles are of limited application in U.K. business.

Perhaps the most advanced and certainly the most technical paper presented is by RICHARD B. MAFFEI of the Massachusetts Institute of Technology. Operational research enthusiasts will find much here of interest in the valuation of uncertainties. MAFFEI sets about tackling the central problem of reducing uncertain uncontrolled situations to risk-controlled situations, but implies that the cost of reducing risks may not always be worthwhile.

ROBERT B. FETTER of Yale University contributes a final paper on the evaluation of uncertainties in inventory control. He develops the idea that the decision game can be played well without necessarily having complete or nearly complete information. Perhaps this is as well to those of us deprived of foresight.

Members who attended the symposium from 9 a.m. to 9 p.m. on Saturday and Sunday, with short breaks for meals, will no doubt welcome the opportunity of digesting these papers at greater leisure. There are ideas and thoughts on this central business problem of capital planning and justification which are worth a few week-ends of thought.

N. R. TRIBBLE

Dynamic Management Decision Games.

JAY R. GREENE and ROGER L. SISSON.

John Wiley and Sons, New York. 83 *pp.* 24*s.*

The use of Business Games in companies' formal training programmes is rapidly becoming an accepted practice. Until recently most of the games available either necessitated the use of a computer or were solely concerned with simulating top management decisions.

Now, however, Messrs. Greene and Sisson have produced a book in which seven different non-computer "functional" management decision games are described, and which may be of considerable help to companies wishing to teach a particular management skill. The games deal with:—

(1) Materials Inventory, based on the standard economic ordering quantity formula.
(2) Personnel assignment, in which (say) accountancy teams of various efficiencies are allocated to specific jobs; a linear programming technique is introduced.
(3) Retailing Department management.
(4) Production scheduling.
(5) Industrial sales.
(6) "Top Operating", a simple model involving selling price, advertising and production.
(7) Market negotiation involving two manufacturing firms and three wholesalers

Each game is presented by giving a background for the instructor, followed by instructions for the player and umpire; tables and sample charts are displayed. There are also two very informative chapters on constructing, directing and modifying Management Games.

H. CROCKER

Anwendungen der Matrizenrechnung auf Wirtschaftliche und Statistiche Probleme.

Physika-Verlag, Würzburg, 1959. 262 *pp. DM*24 (*paper*), 28 (*boards*).

This book consists of a number of brief surveys or papers on matrix theory and its applications, prepared under the auspices of the Deutsche Statistische Gesellschaft. It comprises, first, a brief summary of such elementary theoretical results as will be needed (quite good, but the equivalent could be found in English). Secondly, three articles on the application of matrix theory to production data. These are, in essence, all variants on the input–output type of analysis. The one that goes into the details of a firm's records appears to be breaking new ground, but hardly goes deep enough to show that the approach will be useful. Thirdly, a paper comparing the theoretical bases of linear programming, of input–output analysis and of game theory, and one on the simplex method of solving a linear programming problem: these could be got as well in English. Fourthly, a paper on the maximum quantity of information that can be got from qualitative data, clear and interesting, but in a field in which I am unable to give an informed judgement; a fuller exposition of the author's ideas (A. Adam, *Messen und Regeln in der Betriebswirtschaft*) is to be reviewed in the *Quarterly*. Fifthly, a demonstration that many statistical results can be expressed concisely and elegantly in matrix form. Finally, an account of a few simple computational drills for solving matrix problems, with a large number of references to the literature.

Taken all round, this would appear to be worth having if one can easily read mathematics in German, not otherwise.

T. E. EASTERFIELD

What is Cybernetics?

G. T. GUILBAUD. (*Translated by* VALERIE MACKAY.)

William Heinemann, London, 1959. *vii* + 124 *pp.* 10*s.* 6*d.*

In this delightful little book first published in Paris in 1954, Professor Guilbaud presents a most readable outline of the domain covered by the newly evolving science of cybernetics. Among other things the book is remarkable for its avoidance of the majority of the polysyllabic jargon associated with the subject, as also for its rather complete etymological note on what has elsewhere been described as this science's "ugly but apt title".

After setting the stage with a discussion on the circuitry of systems, feedback and goal-seeking, Professor Guilbaud devotes the major part of the book to

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a most lucid introduction to the theory of information. This ranges from measurement of information, through crosswords and language simulation to the problem of noise and the concept of entropy. The concluding pages give a well-balanced appraisal of the state of cybernetics at the time of first publication.

Operational Research receives a mention (in parentheses on page 121), but readers of the *Quarterly* who seek enlightenment on which of operational research and cybernetics is the chicken and which the egg, may find Professor Guilbaud's definition unsatisfying. It is true that one is slightly the older and that the other has a rather unfortunate odour, but to misquote an earlier writer on noisy systems:—

"Which is which you may never make out
Despite your best endeavour
Of that there is no possible doubt,
No possible, probable shadow of doubt,
No possible doubt whatever."

In fairness it should be remembered that this introduction was written some 6 years ago and that the subject-matter has been developed extensively since that time. It says much for this little book that it can be as warmly recommended now as in the original version.

Valerie MacKay and the publishers are to be congratulated on making Professor Guilbaud's book available in English; the quality of the translation and of the production is uniformly excellent.

G. W. SEARS

The Time Dependent Solution for a Storage Model with Poisson Input.

J. GANI and N. U. PRABHU.

J. Math. Mech. **8**, 5, *pp.* 653–663 (*Sept.* 1959).

The problem discussed is the determination of the probability of emptiness of an infinite dam with Poisson input and a steady continuous release (except, of course, when the dam is empty). The probability of emptiness at time t, $F(O, t)$ is obtained after first finding the probability of first emptiness of the dam, and by following a similar procedure; that of enumerating the ways in which the dam may empty and summing the probabilities for the various ways. The probability of emptiness, in either case, is a function of the initial state. Having calculated the probability of being empty one can then proceed to solve the forward Kolmogorov equation directly. The Kolmogorov equation is given by

$$\frac{\partial}{\partial t} F(z,t) - \frac{\partial}{\partial z} F(z,t) = -\lambda[F(z,t) - F(z-h,t)] \quad (0 \leqslant z < \infty),$$

where h is outflow and λ the mean input in time interval t. The steady state distribution of the dam content can then be derived as the limit of $F(z, t)$ as

$t \to \infty$. When $\lambda h < 1$, in the limit $t \to \infty$, we have

$$F(z,0) = \phi(z) = 1 - \lambda h \sum_{r=0}^{z/h} e^{-\lambda(rh-z)} \frac{\lambda(rh-z)^r}{r!},$$

which is the result of Takács (*Acta Math. Acad. Sci. Hungaricae* **6**, 101 (1955)). The time dependent solution is not available elsewhere.

E. K.

Operations Research, Methods and Problems.

MAURICE SASIENI, ARTHUR YASPAN and LAURENCE FRIEDMAN.

John Wiley and Sons, 1959. 311 *pp.* 82*s.*

This book precisely fills the role which the authors intended, that is as a text on the basic techniques used in Operational Research. It is not a book about operational research, but an applied mathematics book useful for students of at least undergraduate level. One hopes that the information it contains will be known as the basic requirements for the next generation of operational research workers.

There are nine chapters on distinct subjects, with three mathematical appendices. The first chapter on probability gives the usual distributions and their properties. Sampling is practical and introduces Monte Carlo methods; in this chapter there is a numerical misprint on page 50. Inventory and replacement provide useful mathematical solutions to some elementary models. The chapter on waiting lines, however, gives little analysis but quotes results and then uses them to solve various numerical examples. There is a chapter on competitive strategies which gives some games theory. Allocation provides the most useful chapter of the book; it contains problems on assignment, transportation and linear programming giving the simplex method. However, on page 200 a mistake in analysis leads to a feasible instead of an optimal solution. This is apparently convenient since the next ten pages are used to show how to convert a feasible solution to an optimal one, and one wonders whether the mistake was deliberate. The chapter about sequencing is prefaced by the remark that these problems are intrinsically difficult and that no analysis is possible except for some very special cases. No doubt this is almost universally true. Finally Dynamic Programming is introduced with some heavy mathematics.

Unfortunately no discussion of the operational research approach is given in the book and no reference is given of original papers. Thus, all things considered, one may say that this is a good elementary applied mathematics book giving those techniques useful to operational research workers.

R. R. P. JACKSON

Rapid Statistical Calculations.

M. H. QUENOUILLE.

Charles Griffin and Co. Ltd., London, 1959. *xv* + 77 *pp.* 10*s.*

This pocket-sized book satisfies a long-standing need. Rapid methods are scattered throughout statistical literature and there are few connected accounts of their details. Both the format and lay-out of this book are well conceived and the methods described, each with a worked example, cover problems in the following fields: estimation of mean and standard deviation, comparisons involving one or more mean values, comparisons of proportions, tests of goodness of fit, tests for association, estimation of correlation and regression and tests for trend in time series. Some of the methods described are not particularly rapid, being, in fact, the standard methods for the problem concerned. For example the χ^2 test for independence in a $p \times m$ contingency table, or the use of the F distribution to put limits to the ratio of two standard deviations, are both given. In the main, however, the procedures given are shorter than those in common use, gaining their speed at the expense of power and efficiency.

A reading of the book raises some doubts as to the exact class of reader the author had in mind. Many alternative methods are often given for the same or very similar problems. This, combined with the concise nature of much of the explanation and examples that are purely numerical, would suggest that the book is aimed at the professional statistician. The latter, however, is unlikely to need any reminders as to how to calculate a mean or carry out a χ^2 test for goodness of fit. But he would be well placed to appreciate the differences between many of the apparently similar procedures that are put forward. Other scientists might need a little more guidance before being able to carry the volume around as a handbook for immediate use, and they must realize that the methods cannot be applied uncritically. How many statisticians would accept, for instance, the non-significant result of the example for Method 16 at its face value without inquiring whether groups 1 to 3 did not have some affinity and the same for groups 4 and 5? Or again, the utility of Method 22 is somewhat dependent on there being a reasonable number of groups in the distribution examined.

The choice of what methods to include in a book such as this is difficult and must inevitably reflect the author's personal views and experiences. Mention might, however, have been made of Duckworth and Wyatt's quick paired comparison technique (*Operat. Res. Quart.* **9**, 218 (1958)) and Dixon's test for outliers (*Ann. Math. Statist.* **22**, 68 (1951)). A few further points of detail call for comment. The significance limits for Tukey's overlapping test in Method 15 do not agree with the values given by Tukey in *Technometrics*. Also it is common to recommend expected frequencies somewhat greater than 2 for a χ^2 test (4 or 5 is given in *Biometrika Tables for Statisticians*, Vol. I). There are a number of misprints which are, however, unlikely to mislead the serious reader.

Operational research workers will find this a handy volume to possess. It will enable them to make approximate assessments of many problems and put checks on more extensive calculations. The author is to be congratulated on having brought together so many methods within so small a compass. One final suggestion. Many non-statisticians find it difficult to understand why it is that two tests of the same hypothesis on identical data will occasionally give different answers. Perhaps, therefore, in the second edition that this book will surely attain, the author might care to explain rather more fully just what it is that a rapid test can achieve in comparison with the normal test.

P. G. MOORE

Handbook of Automation, Computation and Control. Volume 2—Computers and Data Processing.

Edited by EUGENE M. GRABBE, SIMON RAMO, DEAN E. WOOLDRIDGE.

John Wiley and Sons, New York, 1959. 1033 *pp.* £7.

This central part of a three-volume handbook follows that on Control Fundamentals; the concluding volume will be on Systems and Components.

The thirty-one chapters have been written by no fewer than forty-one contributors, so that there is some overlapping of the subject-matter and a variety of methods of presentation. The material covered is comprehensive and, although the description is of American methods and machines, this handbook should provide an invaluable work of reference to the experienced programmer.

After a chapter on terminology, there is a 270-page section on programming which deals with conventional, automatic and maintenance programming in great detail. An unusual feature of this section is a complete description of the order codes of a number of American computers and one Russian computer. The range varies from the desk computer to the I.B.M. 704 and includes one, two, one plus one, and three address machines. Many forms of automatic programming are also covered.

The following section on the use of digital computers and data processors describes a variety of equipment available and specifies the facilities required when setting up a computer system. Applications described include life insurance, public utility billing, pay-roll, inventory and scheduling and a short chapter on scientific and engineering applications.

The remaining three sections are more technical. The design of digital computers includes chapters on magnetic cores, transistors and logical design. The last two sections on design and application of analogue computers and unusual computer systems should give a broader outlook to the digital computer specialist.

The lay-out of the material in this volume is very good, this facilitating its use as a work of reference. As will be realized, the different subjects dealt with

in a book of this size cover a wide field, but certain parts of it will be of value to anyone interested in computers.

M. C. HANCOCK

Cybernetics and Management.

STAFFORD BEER.

English Universities Press, 1959. 214 *pp.* 25*s.*

The author postulates that the primary long-term objective of Management is the survival of the industrial enterprise which it conducts. It is the purpose of this book to convince scientifically trained and imaginative managers that a machine, using the term in its widest sense, can be constructed continuously to adjust the activities of a firm to its environment in such a manner as to attain this objective in an optimum or near-optimum manner.

The first half of the book ranges over a wide field of formal knowledge, including physics, biology, mathematics, control engineering, philosophy, psychology and economics. From this study an attempt is made to develop a broad, theoretical basis for handling systems which are exceedingly complex because of the wide variety of internal and external factors affecting their behaviour. In a large measure this theory is an adaptation of the principles of feedback loops, the black box and natural selection.

The second half of the book starts by describing two learning machines which have been constructed, and goes on to consider the possibility of intelligence amplifiers, intelligence here being thought of mainly as the power to select. It is the last few chapters which seriously tax the reader's imagination. These deal with machines to reproduce and to improve themselves and they touch, very lightly, upon the form which such machines might take.

The subject-matter of this book is very wide and the treatment in many cases is inevitably superficial, so that the deductions drawn often lack conviction. For instance, feedback is explained, but there is no reference to phase, which may explain why, in the example quoted, positive feedback becomes confused with power amplification. Entropy is described as a natural tendency to orderliness, although the student of thermodynamics may well have learnt to think of it, with Willard Gibbs, as a measure of disorder. As described and used in this book, it is difficult to see how the term helps to clarify the author's purpose. The analogy between the survival of a firm and that of a species through natural selection is pressed too far and again one is left wondering whether there is any useful similarity at all.

Despite such objections, this reviewer readily admits to having thoroughly enjoyed this book. It is written in a lively and readable style and one is easily infected with the author's enthusiasm, if not always convinced by his logic. However, it is to be doubted whether the book will suffice to realize his hope of encouraging industrial management to devote substantial funds to research

into cybernetic machines designed to simulate a whole firm and its environment.

Hard-pressed managers show reluctance to read reports and books unless their interest has already been aroused. In this particular case it may be questioned whether the time is ripe for their interest to be encouraged. The author sees the developments he describes as different from, but as an extension of, operational research. To many engaged in industrial management, the application of the latter is only just becoming acceptable. To claim so much for the ultimate, whatever form that may take, could render the intermediate stages more difficult of accomplishment.

S. W. ADEY

Economic Aspects of Fuel and Power in British Industry—Papers presented by the Manchester Joint Research Council.

Manchester University Press. 217 *pp.* 25*s.*

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The importance of fuel and power in the maintenance of an efficient industrial system, and the fact that great technical changes are taking place within the fuel and power industries themselves, have served to create a new interest in these industries in recent years.

In November 1958 the Manchester Joint Research Council organized a conference at which a number of representative speakers outlined modern technological advances in the supply of suitable fuels and the provision of adequate power to British industry, and dealt with the repercussions of these in the economic field. The papers given to the conference covered oil, coal, gas, nuclear power and electricity, as well as future trends in the chemical and metallurgical industries, and in general fuel technology. The papers, the discussions which followed them and the opening address by Lord Mills, the then Minister of Power, are brought together in this book.

The organizers of the conference hoped to assist those responsible for important industrial decisions to appreciate fully all the factors involved in planning fuel and power developments over the next few years. The same pattern was adopted for the broad treatment of each energy source—a statement of the present supply position with a survey of the main end-uses, the technical advantages involved and likely future developments, followed by a discussion open to all delegates.

The book is a valuable starting-point for the student of fuel and power problems in this country, and has some historical worth for its compact presentation of the views held by the fuel and power industries in 1958. Much of the contents, however, as the Council admit, is vulnerable to the test of time. The authors in the main, mindful of Lord Mills's comment that "the wisdom of one year may be almost irrelevant the next", avoided long-term predictions. Even in the short-term events move quickly. Thus, since the conference sat the shadow of the Suez crisis has receded; "the more than promising results of

recent work in the Sahara and other parts of Africa" have been translated into oil production on a large scale; the "Revised Plan for Coal" has been produced; railway dieselization has been accelerated and the nuclear programme re-phased. These developments colour the background of the conference, but do not detract from the value of this record of its work, nor are they in conflict with the general expectation "of a more rapid increase in the use of oil reflecting continued expansion of road transport, the re-equipment of the railways, the continuance of recent trends in domestic heating, and the switch from solid fuel to oil in those industrial and commercial uses where it has special attractions".

The discussion periods, in which the claims of one industry came under fire from rival delegates, have particular interest. It is noteworthy that the conference accepted the Minister's view of a national fuel policy, as meaning the promotion of "the supply and use of fuel and power on that scale and in that pattern, which will make the best use of our resources", and agreed that there must be no mandatory instruction to the consumer as to which fuel he should use. In the words of one delegate, "if we allow ourselves to be manœuvred into a position where others determine what we have to do, then we have only ourselves to blame".

One drawback from the point of view of the general reader—or for that matter the industrialist—of the method of presentation of papers at the conference is that too much of the discussion is within the framework of a particular industry and too little is devoted to a joint appraisal of those fuel and power usages, where the claims of the competing industries directly conflict. Thus we are told in relation to space heating that "there is no agreed method of assessing the comparative economics of different types of heating system" —yet it is just this type of decision which the industrialist is called upon to make.

This book is, however, easily readable and, presenting in compact form the industrial applications of fuel and power in this country, is worthy of study by those whose job involves an understanding of these problems.

M. E. WEINSTEIN

News and Notes

New Full Members of the Society

R. B. Davies—Central Electricity Generating Board, Cardiff.
M. Dizy—A.E.I. Ltd., Manchester.
D. J. Finch—British Transport Commission, London.
J. S. James—A. E. Reed & Co. Ltd., Maidstone.
A. J. Mayne—University of Leeds, Leeds.
H. V. Murphy—U. K. Atomic Energy Authority, Warrington.
J. H. Nicol—National Coal Board, London.
E. Page—The Steel Company of Wales Ltd., Port Talbot.
W. E. Prior—GKN Group Research Laboratory, Wolverhampton.

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Meeting

The Seventh International Meeting of the Institute of Management Sciences will be held at the Hotel Roosevelt, New York City, 20–22 October, 1960. Further information may be obtained from Mr. JAMES TOWNSEND, Union Carbide Corporation, 270 Park Avenue, New York 17, New York.

Activities of the Arbeitskreis für Operational Research (AKOR)

Seventy operational research workers from Austria, Belgium, France, Germany and Switzerland attended the first course on Operational Research organized by AKOR at Arnsberg, Sauerland, Germany, from 16 to 25 May, 1960, under the chairmanship of AKOR's president, Mr. HELMUT KREGELOH.

The course heard lectures from Prof. R. SHEPHARD, University of California, on linear and dynamic programming; and from Prof. LINDER, Geneva, and Prof. WALTER, Darmstadt (both members of AKOR from the start of this organization in 1956) on planning of experiments and on the technological progress in the field of electronic computers. Other AKOR members spoke on operational research problems in their respective industrial firms and institutions such as transport, allocation and queueing problems in iron and steel, coal-mining, chemical and electrotechnical industries, and in traffic.

Mr. H. KREGELOH in his introductory speech and when summing up the results of the course underlined AKOR's primary goal consisting of the exchange of essential experience to promote operational research in Germany where the importance of the new methods is more and more recognized. The lectures followed by intensive discussions proved this first course on operational research in Germany to be a very successful one.

LONDON SCHOOL OF ECONOMICS

Second Short Course on Operational Research

A further two-weeks' course was held at the London School of Economics at Easter 1960. There were over thirty participants from a wide range of industrial organizations and government departments. The main purpose of the course was to bring active, and in some cases potential, workers in operational research up to date as regards current techniques and recent developments. Most of the lectures were followed by lively discussions in the lecture room and outside. The course was thought to have been successful in stimulating interest and facilitating private contacts among practitioners. Among the more encouraging features was the evident maturity and professional attitude, even of the younger participants who no longer felt it necessary to have the basic principles and grounding of operational research brought out, but were anxious to increase the scope of their own contributions to it.

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Obituaries

Dr. Cecil Gordon, O.B.E.

Older members of the Society will have learnt with regret of the death of Dr. Gordon in March of this year at the age of 54.

Cecil Gordon was of South African origin. After taking the degree of M.Sc. at the University of Cape Town, he came to England and made a career in genetics, taking his degree of Ph.D. at the University of London.

As with many other natural scientists, the war drew him into operational research. At Coastal Command, the Operational Research Section of which he joined in 1942, and subsequently at Air Ministry, he was almost wholly concerned with problems dealing with the efficient disposition of manpower in relation to aircraft.

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An enquiry, from outside Coastal Command, into the apparently low utilization of certain types of aircraft, led him to investigate the organization for maintaining the aircraft and for deciding the use of them when serviceable. A realistic mathematical model would be of tremendous complexity, imposing on to a multiple interference problem a number of extraneous random disturbances. A drastically simplified model, using hardly more than fourth-form algebra, was, however, sufficient to lay bare the basic skeleton of the problem, revealing both a workable criterion of maintenance efficiency and the factors that were most worth examining to allow efficiency to be raised.

On the first of these, it became clear that the criterion of efficiency then current had become, in fact, a cause of inefficiency, though it had been the right one to use in pre-war years. On the second, the seminal nature of the work was shown by the large number of studies of effects of maintenance, effect of use on aircraft performance, balance of trades and methods of reporting aircraft states, that arose directly out of it. The actual mathematics was, in fact, the least interesting thing about the work, even though a number of interesting and important mathematical problems were left over for future solution. The team on this work were fortunate in being allowed by the Command to try out their ideas, when they were able to show a striking increase in the amount of flying done by two experimentally controlled squadrons. It was for this work that Cecil Gordon was later awarded the O.B.E.

In 1946 he moved to the Board of Trade as head of the Special Research Unit created, apparently, in the hope that operational research could be done on the Board's activities. In fact, the Unit found itself making studies of aspects of industry that involved both economic and technical factors; the atmosphere of the Board was not at that time sympathetic to attempts at forward planning, or to studies of the effects of its activities. The Unit never really established itself,

though some successful pieces of work suggested that more might have come out of it had either the members of the Unit been more sympathetic in their dealings with the officials of the Board, or vice versa. After his departure in 1948, the Unit faded away (as had done a similar unit set up at the Board just after the First World War).

After leaving the Board, he returned to academic work, first on genetics and later on social medicine, at the University of Edinburgh. His recent work was on the employment of elderly people, to which he brought something of an operational research outlook in his examination of the structure of the whole context of their getting and holding jobs.

He was among the earliest members of the Operational Research Club, which later grew into the present Society.

The immediately striking thing about him was his enthusiasm, coupled with a mind that was, at any period, almost exclusively concerned with his current interest. He was of enormous, if sometimes uncritical, fertility: any chance observation or remark could be the stimulus to a flow of speculations, hypotheses, qualifications, rebuttals. Possibly because of his biological training, he took a very eclectic attitude to techniques, pressing into service simple description, verbal argument, analogy, simple or complicated mathematics, as the problem required. For him, the essence of operational research was the finding out of the structure of a problem and of its real-life setting; mathematical or other tools were things to be picked up when required where they were available.

His highly polemic style of argument sometimes made for difficult relations with the officials with whom he dealt; but, as long as one took it in the spirit in which it was meant, he was an admirable person to work with, highly stimulating to work for, and always keenly concerned for the well-being and advancement of his staff.

T. E. EASTERFIELD

Dr. Miriam Gilbert

We regret to report that Dr. Miriam Gilbert, a full member of the Society, was killed in a gliding accident in Austria on 16 June. Miss Gilbert was with the Ministry of Transport and Civil Aviation.

The Aim and Policy of the Operational Research Quarterly

AIM

The *Operational Research Quarterly*, which is the journal of the Operational Research Society, is a means by which the Society records, makes known and advances the theory and practice of operational research. The Journal is intended not only for practitioners of operational research, but also for those who wish to learn more of the subject.

POLICY

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Publication of Papers

(i) Papers presented at ordinary meetings of the Society will in general be published in full, without the ensuing discussion.

(ii) The submission to the Editor of original papers which have not been previously presented to the Society is encouraged, particularly those referring to:
   (a) Operational research case studies.
   (b) Derivations of mathematical and other scientific theories provided they have already, or are thought likely to have in the future, a definite operational research application.
   (c) Review papers outlining and discussing the theory applicable to particular classes of problem found in operational research.

(iii) The publication of papers is subject to the following conditions:
   (a) Publication is at the discretion of the Editor who will maintain standards by submitting manuscripts to at least two referees.
   (b) Authors will not be paid for papers but on publication they will receive twenty-five free reprints of the paper.
   (c) All papers should be preceded by a summary which would make a suitable abstract of the paper. This should indicate the scope of the paper, the main contents and the conclusions.

Abstracts

Abstracts will be commissioned and published of papers which have been published elsewhere over as wide a range of British and foreign journals as is practicable.

Book Reviews

Reviews of books which may be useful to workers in operational research will be commissioned and published.

Aim and Policy

Correspondence

Letters to the Editor are welcomed and will be published at his discretion.

News

Items of news at home and abroad relating to operational research will be published.

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The Place of Operational Research in the Development of Modern Society*

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FOR some years the branch of study known as operational research has been making rapid strides, and, like so many other swiftly developing sciences or techniques, it is found somewhat baffling by the layman and even the specialist. Instances of its work or the mathematical curiosities it has produced stagger their imagination but leave them at a loss to assess the place of this new study in the development of society to-day.

The present article is an attempt to analyse operational research in this context, as the best way of elucidating its significance and outlining its future. The two points to be demonstrated are: (a) that operational research is not the fruit of spontaneous generation, but the outcome of a slow process extending over several decades, and (b) that the phenomenon of operational research is closely bound up with other aspects of modern society.

Finally, I shall try to give an idea, in broad outline, of the impact that operational research may have on the future development of society.

Everyone, or nearly everyone, regards operational research as a wartime creation of the Anglo-Saxon countries, as a discovery made by academic specialists working with the army. But to take such a claim at its face value is tantamount to dating the emergence of a particular outlook from the moment when its originality is so clearly realized that people think fit to give it a name. In other words, it means discounting the long period of maturation that went before.

To go back further than the end of the nineteenth century, the spirit of operational research may be said to be discernible even in Taylor's writings. Taylor's first papers were concerned with metal cutting at Bethlehem Steel.

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He discussed all the possible variations in the speed and depth of the cut and in the shape of the cutting tool, and endeavoured to determine the best cutting methods either by rule of thumb or with the help of mathematical formulas prepared by specialists.

During the 1914–18 war Lanchester (whose name was later attached to a prize in operational research) worked out equations for military operations, and some years ago these equations proved to be a satisfactory means of assessing the evolution of the battle of Okinawa.

At about the same time Pareto, working in a field having no apparent relation whatever to economics, gave final formulation to the meaning of the theory of the economic optimum which was to assume so much importance in France 40 years later.

The second decade of the present century saw the birth of commercial studies in the United States of America. The first market study is said to have been carried out somewhere about 1910–11 by the Curtiss Publishing Company, a magazine publishing house. Feeling itself at a loss for arguments to put over its advertisements and having just bought up a new agricultural magazine, *The Country Gentleman*, it decided to study the market for this periodical by questioning farmers and the manufacturers of farm machinery and products about their needs and wishes. This first study gave the company a great fillip and was very well received by its readership.

And so, on all sides, definite trends began to appear independently and spread during the first half of the century until they gradually merged into a recognized body of thought after the second world war.

The year 1930 was marked by the establishment of the Econometric Society, whose journal *Econometrica* was largely instrumental in the development of modern economy. At the same time the growth of national accountancy opened up a new field of action for market research, survey and sampling techniques were perfected, and the calculation of probabilities and statistics assumed more definite form and ceased to be a field of knowledge confined to a small group of leading specialists.

But in this, as in so many other fields, the war was to hasten progress. In the United Kingdom, and later in the United States, the armed forces developed operational research and operations research units, whose principal forms of activity are too familiar to need recalling here.

In France, the nationalization of many branches of industry, coupled with Mr. Allais's efforts to popularize the theory of the economic optimum, prompted growing numbers of engineers to ponder on the management of large industrial concerns and to study the economic aspects of production, pricing and investment. The first step towards a unified approach to such problems came with the establishment of the Société Française de Recherche Opérationnelle; certain members of the founding group were mainly influenced by Anglo-Saxon research, others by French research.

At present, these various currents of thought that have sprung up since 1920 are merging together and modern management techniques are taking final shape. While it must be admitted that the terminology used to define them is still very confused, terms such as econometry, applied statistics, marketing, operational research and management science may be said to denote approximate areas around which research or its applications have concentrated rather than clearly definable corpuses of study.

If we wish nevertheless to define the position of operational research in relation to other branches of learning, it would perhaps be best to adopt the following principle. Scientific thought has always followed a two-way course of development—that of pure or fundamental science, the aim of which is to perfect models enabling us to describe the concatenation of actual phenomena, and that of applied science (or technology), which helps us to define coherent practical objectives and suitable means of achieving them. The fundamental sciences and branches of technology have their points of contact, of course, since each branch of pure science gives rise to one or more branches of technology, while each branch of technology is in turn related to one or more of the fundamental sciences. We are now quite accustomed to this situation in the physical sciences, and are so well aware of the relative status of technology and science that we are apt to forget that some of the branches of technology began their development in advance of the relevant branch of pure science and to some extent independently of it. We are now witnessing the same process in the new sciences. In the case of pure science, scientific disciplines covering categories of phenomena are being built up, the most advanced of them being economics and psychology, whereas others such as sociology, administrative science, political science, etc., are only just beginning to come into existence. Until recently, these sciences had not given rise to any branch of technology. In other words, the counterpart to scientific thought in the field of applied science was a vast "no man's land", the most accessible part of which was naturally invaded by different armies having the same aims but different backgrounds. Economic science has thus given birth to a technological branch that is sometimes known as econometry. In the United States and the United Kingdom, the pioneers in operational research have been the physicists, mathematicians, chemists and in some cases the biologists. Their armies are still encamped, pell-mell, on the same field, but having once taken up their positions, they have become increasingly aware of the importance of economic factors.

In time, the term "operational research" may possibly be discarded, should the study fail to establish itself as a field in its own right once the exact relationship between the different branches of technology and the fundamental sciences has been finally determined.

A question often brought up in this connection is the relationship between operational research, organization and the automatic processing of information.

Organization would seem, basically, to be an administrative "pre-technique" in that it was built up before administrative science and depends more on intellectual honesty and common sense applied to the observation of facts than on scientific method in the strict sense of the term. Organization is therefore largely situated in our "no man's land", in one of the areas still beyond the reach of scientific method. In the case of the automatic processing of information, on the other hand, the field covers the whole range of problems arising from the use of automatic computers. The latter, incidentally, are not at present used for more than one in every five or ten operational research studies.

The term "operational research" thus covers a broad and deep-running stream of thought which has been gradually formed over the past fifty years and reflects the progressive advance of science into the realm of social problems.

Once operational research has been assigned its proper place in the stream of thought of which it is only one element, it is far easier to see how far it mirrors present-day collective thinking, the main features of which are readily discernible.

1. Scientific thought still has considerable power of expansion. The ideologies which have grown out of it are still dominant, and large numbers of men engaged in every field of research are prepared to renew each day, as a working hypothesis, their act of faith in the possibility of building up concepts and systems whereby all phenomena can be apprehended. The success of this method in physics, chemistry and biology is an obvious argument for applying it to the study of economic and social phenomena.

Yet this expansion of scientific thought has had to be fought for. For a long time it was halted by the lack of understanding paradoxically shown by two different classes of people.

For the layman, the scientific method seemed to amount to no more than plain intellectual honesty—an attitude that is responsible for the emergence of such inadequate expressions as the science of law, political science, and so on. For those with scientific backgrounds, the scientific method seemed inseparable from traditional physics, the only field in which disagreement between theory and fact was thought to be comparatively negligible by human standards. It was gradually realized that science was a harder task-master than plain intellectual honesty and that rather than being tied to any single discipline it involved the application in all cases of a single method—the construction of operative concepts and the elaboration of models. It was this realization, for example, that led to the discovery, from the conceptual standpoint, that just as in physics mass is a constant reflecting repeatable relationship, so the phoneme of the original Indo-European language family in fact represents the point of contact between clearly defined phonemes of languages such as Greek, Old Slavonic, Lithuanian, Persian, Sanskrit, etc. This definition of concepts has its repercussions on vocabulary, and produces

a progressive tendency to replace terms whose meaning is enlarged by refinements and ramifications of connotation by terms merely serving as conceptual symbols. At the same time, it was realized that theory cannot be equated with reality, and the term "concept" was replaced on occasion by the term "model", which places greater stress on the construction of a network of relations between concepts. It should now be possible to demonstrate the underlying analogy between the models of the different sciences. Models may be either static or dynamic, cyclic or cumulative; they may consist of functional or conjectural relations or show how the part fits into the whole. What difference is there between "simulation", the value of which has been emphasized by operational research, and the practice of making small-scale models? It might almost be said that operational research came into being when the first grave doubts arose over the meaning and general applicability of the scientific method.

2. The very growth of technology based on twentieth-century scientific discoveries has made it imperative to apply scientific methods to the field of management. Production units are daily increasing in size and complexity; organizational problems are becoming more and more intricate; and important decisions now involve a growing number of variables, which are difficult to control. Whereas, a few decades ago, factory supervision was a simple matter, often concerned solely with the quality of goods, technological progress has made it both possible and necessary to provide accurate measurements of operations and keep a continuous record of their progress. Every industrial concern has permanently at its disposal a mass of detailed statistical information regarding its output, sales, stocks, machine repairs, etc., and is in a position to initiate new schemes to improve its management. Facts and figures are available for increasing profits by several per cent more than could be achieved by relying only on intuition and experience. Costly though it may be, the research entailed becomes an economic proposition where the sums involved (as in large production units) are considerable and where the rather rigid organization of the units (if large) implies the establishment of production standards for fairly long periods. The type of engineer who begins the morning by allocating the day's work in the best possible way is dying out. Science calls for scientific methods. The coexistence of technological practices and that amalgam of traditional formulas and intuition which prevails in the management of industrial concerns, government departments and the like is only possible in the areas adjacent to a borderline forever changing.

3. The effect of these purely technical factors is accentuated by the crisis of Western civilization. The criticism levelled at capitalism has undermined the confidence of the present generation in the efficacy of an economic system in which decisions are decentralized. We are becoming more and more intent on controlling economic phenomena. Governments make it their aim to prevent fluctuations in general conditions, ensure balanced economic growth and see that correct choices are made in the allocation of resources.

Again, there is the desire to influence demographic phenomena by trying to raise or lower the birth-rate as the case may be; while the psychologists, for their part, try to devise methods of giving individuals better guidance or preparation for life. And so, in all fields, the present-day world is in quest of rational patterns of behaviour. Intellectual leaders are exercised by the problem of how to run society, and the whole subject is a very anxious one. Conscious as we are that past civilizations lived only for a limited span, our aspiration is to dominate the course of history, and so we are constantly scrutinizing other civilizations as reflected in their religious and administrative institutions, their ethics and their art, in search of an answer to our own problems. And it is paradoxical, in this connection, that operational research has been developed by the same society that gave birth to sociological criticism and to the great reconstructions of world history by Toynbee, Spengler and so many others.

4. From the social point of view, the development of modern management techniques is also bound up with the growth of the social stratum (the term "class" would be straining the facts) consisting of those people whom Burnham describes as managers; they are for the most part—in France at any rate—the senior officials or engineers who control the big administrative or industrial units. In the nineteenth century, a predominant role was played in small- and even large-scale industry in France by the *patron* who, after completing his secondary schooling, was trained by his father in the business he would one day run. The main task of engineers at that time, therefore, was to formulate manufacturing processes. The growth of joint stock companies, the concentration of industry and the desire for power gradually induced the engineering graduates of the French professional high schools to switch from purely technical to managerial posts. As they have all gone through much the same training, they now form a very homogeneous group with little interest in political power but playing an increasingly decisive part in economic expansion.

The scientific knowledge they have acquired before or during their career makes them very favourable, relatively speaking, to the principles of operational research. At the same time the many opportunities which present-day legislation affords senior civil servants to play a part in economic life are drawing them into industry and encouraging them to overhaul traditional methods of management. In this connection, we need only mention the influence exercised by the Commissariat Général du Plan.

5. It must also be admitted that in certain cases the development of operational research is accompanied by that loss of initiative and zest for action which is an unfortunate feature of our Western societies. In France much of the time of our administrators is taken up with undoing the work of their French or foreign colleagues, or in out-manoeuvring particular social groups. The system of commissions and sub-commissions, coupled with the mental agility of the graduates of the French professional high schools only adds

to the intensity of the endless competition for influence. We are not so fond of taking risks as we used to be, and before any decision is taken it is carefully dissected and examined from every angle. Operational research is useful in this respect because it gives those who have to make decisions the support and guarantee they need, but on no account should it become an instrument in the hands of those who are merely trying to use science to defend their own interests or place further obstructions in the way of decision-making, under the pretext of "studying" a particular question. In support of the point argued above is the fact that—so far as we can judge from the scrappy information available to us—the Union of Soviet Socialist Republics, which has a completely planned economy, lags behind in matters relating to applied economics, operational research and theories of decision-making.

The above survey is, of course, a mere summary of the facts, which need to be subjected to a more searching analysis. It does, however, give a general idea of how operational research ties up with other developments of the modern world.

It explains to us how we, following the example of the Egyptians who in all probability developed two-dimensional mensuration under the pressure of agricultural requirements, are in turn constructing a mathematical system that will meet our needs—a novel one that is concerned with programming and probability factors. And we are quite justified in speaking of the mathematics of programming, for our aim in very many cases is to find the combination of decisions (and hence the values of certain variables) that will enable us to derive the maximum profit from a particular operation, within the limitations of our possibilities (i.e. limits imposed by the pressures on the variables). What makes this problem even more complicated is the fact that we often have to take a succession of decisions depending on the time factor. In many cases, also, variables can be given certain values only (for instance, the size of a thermal power plant is bound to be an arbitrary choice, for the present state of technology dictates that power plants must consist of standard sets). This gives rise to combinative mathematics—a branch of study which is still in its infancy. At the opposite end of the pole comes probability theory, covering research into rational behaviour in the presence of probability factors occurring with a given frequency. Instances are problems of supplies, queueing, regulation of production and stock-piling. This shows how operational research can impinge even on mathematics.

Approaching the question in more general terms let us now consider the potential impact of operational research on the development of society and contemporary thought and try to forecast the way it will affect technical manufacturing processes, the organization of individual undertakings, administrative organization, scientific thinking.

Operational research will facilitate the transformation of manufacturing processes, and in that respect its development will be closely linked with that

of the automatic processing of information, on the one hand, and of automation proper, on the other.

There is a tendency among laymen at present to confuse operational research and automatic processing of information although as far as operational research is concerned electronic computers are only one of the means of processing the numerical data which it works on: many of the operational research problems can be solved numerically by hand or with the aid of office machines. Electronic computers are not yet so easy to use, and that form of mental laziness which consists in giving the machine inadequately formulated problems to solve needs to be fought against. But a change in the relations between operational research and the automatic processing of information is bound to take place for the following reasons:

1. The latter will increase the investigative means at the former's disposal by permitting the continuous recording of important data.

2. Operational research workers, with their added experience, will be able to co-operate more effectively with scientific computation experts.

3. Computing machines will become progressively easier to work as time goes on. The idea has already been mooted of formulating programmes permitting the use of a logical or mathematical language which comes as close as possible to ordinary language. The efforts in that direction have given rise to interpretative programmes and auto-programmes. The interpretative programmes (e.g. Flair and Flex for the IBM 650) are recorded on the machine whenever it is a case of executing a programme set down in logical language, and the instructions are then interpreted as and when they occur. The purpose of the auto-programmes (AP2 for the BULL AET, Fortran for the 704, Paso for the 650) on the other hand, is to produce the machine-language version of a programme which is fairly easy to construct and at the same time make a certain saving in computation time. This procedure results in a very good utilization coefficient for the machine despite the apparent loss of time represented by the phase during which the final programme has to be established from its "auto-programmed" version.

4. Computers will enable us to solve problems beyond our present practical capacity. The number of memories in a computer and the speed of computing are very important factors as far as combinative problems of all kinds are concerned. In point of fact, there is a general absence of mathematical methods applicable to such problems whereby we could determine the optimum solution by successive approximations, and we have often no option but to review all the possible solutions and determine which one gives the highest value to the function to be maximized. It is true that this review is conducted in an intelligent manner and may result in cutting computation time to a tenth or even a hundredth, depending on how it is done, but the fact remains that the number of solutions to be reviewed is extremely high. The perfecting of computers means a considerable increase in this connection in the progress of operational research.

5. At the same time, operational research studies will themselves be an aid in determining optimum systems for information processing. They will be useful in establishing forecasts of the evolution in time of the volume of information to be processed, and enable breakdown costs and the economic margins for a variety of technical solutions to be determined.

It is also easy to forecast how operational research and automation proper will affect each other. The latter is simply inconceivable in the absence of studies on such subjects as the number of safety devices to be installed, the response time of servo-mechanisms, the arrangement of factories, the size of production units, and so on. The case of oil refineries—or, to take a broader category, of distillation plants—sufficiently proves this point. Moreover, the operational research problems raised by automated plants will be better formulated, since the numerical data will be more precise, the characteristic values more constant and the relations between them more stable, thus facilitating the establishment of mathematical models. The increased size of production units again will be a factor enhancing the profitability of operational research studies, for whereas the profit accruing from a study is more or less proportionate to the size of the unit, its cost increases less rapidly than the latter.

The effect of operational research and related techniques on the organizational structure of undertakings will be no less profound. The chain of relationships in the old type of undertaking was mainly hierarchical, with information and instructions passing from top to bottom, and the directors were men qualified to handle problems by their combination of administrative and technical ability. The diversification of techniques, together with the new thinking of which operational research forms part, have been contributory factors in the advent, succeeded by the steady development, of a functional chain of relationships conveying information and advice of an essentially technical nature. The present tendency is for the modern director—the executive, in the Anglo-Saxon countries—to have at his command a general staff of functional assistants whose importance increases the higher the hierarchical level. These staff groups are in constant touch with each other, and the effect is to render the hierarchies somewhat looser and less rigid. At the same time the development of inter-communication promotes a collective approach to the work. The short-circuiting of individual steps in the hierarchical ladder, at one time heavily frowned upon, is no longer taken so seriously, and there are many problems today which are now studied by joint bodies consisting of representatives of the hierarchy and of the various functional services. The future development of operational research will be even more strongly marked by an increase in the size of these functional general staffs, whether they take the form of internal services or—for particular problems—of outside consultants. The development of consultant firms is significant in this respect. Until a few years ago there was a clear-cut distinction between engineering research firms—major undertakings equipped in many cases with laboratories and employing

large staffs of specialists in various fields—and labour organization consultants' practices headed by a strong personality assisted by a small body of engineers always liable to launch out on their own. In those days, the consulting engineer was essentially a man whose individual qualities were what counted. The advent of operational research and the development of modern management techniques, on the other hand, will probably give rise to consultants' practices which have a much closer resemblance to engineering research firms. Their staffs will include a wide range of specialists, and their efficiency will depend on the volume of resources at their command and the level of competence of their teams as a whole.

It is probable that relations between operational research and labour organizations will also become closer, for it can legitimately be expected that the next twenty years will see the birth of administrative science in the true sense of the word, and based on experimental studies covering the transmission of information in organizations, the implications of individual flow-charts and the various aspects of centralization and decentralization. Operational research will be able at that stage to give real service in determining the most appropriate administrative structures.

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But it would be dangerous to deduce from the foregoing that the importance of the director will diminish. The object of research work is to help him to reach a decision, and its conclusions are not binding on him: it is for him to make the decisions in the light of the arithmetically undefinable factors inherent in the problem. Herein lies the element of risk which is the essence of life itself. There will thus be an increasing differentiation between the man who makes the study and the man who makes the decisions—a differentiation conditioned by their personal aptitudes or dissimilarity in psychological make-up. We cannot have good operational research in the absence of executives, and one of the latters' most difficult tasks will be to achieve a proper balance between studies and decisions, for an excess of studies may destroy the spirit of initiative and the ability to reach quick decisions, while over-hastiness in making decisions may lead to very costly economic blunders. The fact that the spirit of initiative is not always so marked as it was in the nineteenth century will make it all the more difficult to strike that balance.

Operational research may have certain specific effects on administration apart from those just described. By enlarging the possibilities of making rational decisions in respect of integrated industrial units, it will whet the taste for planning—whether we like it or not—and make it easier for the State to take control of the economy. Leading officials in all countries are deeply concerned at present at the poverty of the means at their disposal for analysing the tremendous repercussions that public investments are having on the development of their national economies. Many of the economists have been trying for the past decade to transfer economic cost calculation from the factory level to the community level. Where a small amount of gas is supplied to a given area, for example, the price is high and the only purchasers are those

consumers who find its use profitable. If the supply is increased, the price falls and more and more sections of consumers are reached who are prepared to pay the correspondingly lower prices.

It is only the marginal consumers who derive no more than a minimum benefit from purchasing the gas: the first batch of consumers, who had been prepared to pay a higher price, find its purchase far from unprofitable. A start is also being made at present on the study of urban development costs in the hope of obtaining a more rational conception in a few years' time of the problems of area development, taking account of the differences in incomes that would result from the establishment of undertakings in different localities and the differences in cost in developing those localities. All these studies have the effect of bringing out the divergences between public and private interest which are increasingly leading the State—rightly or wrongly—to take action in the matter. At the same time, however, operational research can help to render State control less blind by providing the powers that be with better information about the consequences of State action, and it can also help administrators to develop a regular philosophy of State action which would make administrative decision less arbitrary and give officials a keener awareness of the public interest. Operational research specialists should beware, in this connection, of allowing their studies to become two-edged weapons in the interminable discussions of experts and the struggles for influence. They should refuse to conduct studies in cases where the numerical data is selected in such a way as to bias the result in a certain predetermined direction, and where the purpose of the study is to help to justify a pre-established political line in the eyes of outside parties. The results of operational research studies are often so closely linked with the data, in fact, that only the specialist can detect the slight "twist" which causes the findings to be modified. Nor does operational research stand to gain if it has to rely for its development on a refusal to accept responsibility.

Lastly, the future relations that will develop between operational research and scientific thinking can be easily descried:

1. Operational research will give rise to a tremendous development in new branches of mathematics, and especially of combinative mathematics and probability theory. Typical examples are near at hand. It has already given rise during the past few years to the theories of linear programmes (including linear programmes in integers in relation to determinants) as well as of dynamic programmes, and to the theory of graphs, the possibilities of which are only now beginning to become apparent. In the field of probability theory, meanwhile, progress has been made in research on conjectural processes, the functions of statistical decisions and the laws of probability.

2. Operational research will also be of help in the formulation of effective methods of numerical calculation.

3. It will act as a nursery for developing those of the pure sciences which stand in the same relationship to it as physics and chemistry do to chemical

engineering; namely, economic science, administrative science, political science, sociology and psychology (in some respects) and military science. To mention only one or two points as far as economic science is concerned, it can be said that substantial progress has been made during the past decade in its application to industrial management but that much remains to be done—as tasks for the future—in studying economic interrelations, urban development costs, regional and national development problems, and guidance in group decision-making. The work in all these fields will be done in close symbiosis with operational research, which means that educators and research experts will be far more intimately linked with national economic life than in the past.

But it would be wrong to imagine that this development will be altogether plain sailing. Operational research will have to face difficulties and will meet with opposition and setbacks. It will have to begin, in fact, by learning humility. Unlike many of the scientific disciplines, it was born in a spirit of pride, and many of those who are not expert in the subject are still asking whether it can solve all problems. Time itself will reduce everything to its due proportions, but at the cost of temporary disappointments. The good work will be separated from the bad by a process of decantation; and there will be periods of stagnation, like the one that now seems to have opened following the amazing creative effort of the past few years. Some of the sectors now included in operational research will become so familiar that they will form part of every engineer's normal equipment. The rest of the field will probably be broken down into narrower but still related specialities, and this again will be a source of sometimes fruitful and sometimes barren discussion. The term "operational research" may even disappear from current language and the work itself be absorbed by various branches of technology closely allied with some of the scientific disciplines.

The issue, nevertheless, is not in doubt, for operational research is not an isolated or accidental phenomenon but the manifestation of a state of mind stemming from modern group psychology. The Western European countries, which have played an always creditable part in the development of operational research, will make a decisive contribution towards its future progress, for our engineers, though they may lack interest in technical details as such, often combine that characteristic with a sound general education and a good knowledge of mathematics.

Contact with operational research may even, in fact, be an excellent school of training for them. It teaches them to be painstaking in elaborating numerical data; to seek out the valid and relevant hypotheses in every problem; to keep their minds always alert; and to strike a fruitful balance between a pragmatism completely dependent on reality and an urge to abstract thinking which loses sight of the facts. The prospects then are fairly favourable; and it now remains for our generation to draw the necessary conclusions as far as theoretical research, practical application and instruction alike are concerned.

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A Problem in Vehicle Fuel Consumption

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The application is described of two alternative methods of solution, employing Dynamic Programming and Lagrange's Multipliers, respectively, to a problem of calculating the minimum fuel consumption of a bus fleet. In essence the problem is how to divide up the time scheduled for a complete journey amongst the intermediate sections in order to minimize the total fuel consumption.

CONTEXT OF THE PROBLEM

IN 1953 a problem concerning the optimization of bus fuel consumption was dealt with by the London Transport Executive Operational Research Section. Two independent methods of solution were found. One employed Lagrange's Multipliers and the other, although it was not recognized as such at the time, was an application of Dynamic Programming techniques. In view of the possible occurrence of similar problems in other fields, it is thought that readers of the *Quarterly* might be interested to hear of this application.

The work was part of a long-term project being carried out into bus fuel consumption. Several aspects were being investigated, including the effect of such factors as vehicle design, maintenance procedures and type of fuel and lubricating oil, but this particular study concerned the effect on fuel consumption of the manner in which the vehicle was driven, i.e. rates of acceleration and braking, speeds at which gear changes are made, top speed reached, etc. Certain driving practices, such as excessively high revving in the gears and intermittent use of the accelerator, were known or suspected to be wasteful of fuel, but the extent to which such practices were prevalent over the fleet as a whole and their effect on fuel consumption was not known quantitatively. Experiments carried out in the U.S.A. had suggested that the effect of inefficient driving practices could be to increase fuel consumption by as much as a quarter.

Before embarking on detailed investigation of all possible variations in driving practice, it was decided, first of all, to attempt to calculate what would be the minimum possible fleet fuel consumption if driving techniques were at all times optimal. By comparison with the actual fleet consumption it would then be possible to assess the scope for improvement and to consider to what extent it was worth continuing the investigation further.

In essence the journey of a bus may be regarded as a series of short runs between stops which are made for a variety of reasons, including picking up and setting down passengers, street intersections, pedestrian crossings and general traffic reasons. The incidence of stops in practice may be determined by direct observation on buses in service and expressed as a frequency distribution for inter-stop distances of various lengths. (By inter-stop distance is meant the run between two consecutive stops.)

* Now with British Transport Commission.
† Now with Central Electricity Generating Board.

By means of controlled experiments on a test track the fuel consumption of a bus between two adjacent stops using any specified method of driving may be determined. By this means it is possible to find the most economical method of driving and to express the minimum fuel consumption for an inter-stop run as a function of the distance and of the time taken. Over the practical range of operation, fuel consumption for a given distance increases as the time allowed is decreased. The reason is, of course, that reduction in time necessitates acceleration to a higher speed.

It should be mentioned here that before the problem could be reduced to its essentials, allowances had to be made for several other factors, including the time spent and fuel used at stops whilst the engine is idling, and the effect of vehicle weight.

The problem remaining, for which the specific techniques described later were required, is as follows.

> If a bus makes a journey consisting of a series of short inter-stop runs, how should the total time allowed by the schedule for the whole journey from one terminus to the other be divided up between the inter-stop sections in order to minimize the total fuel consumption?

The detailed treatment of the optimization problem by the two methods of Dynamic Programming and Lagrange's Multipliers is given in later paragraphs of this paper. As the fuel consumption curves were determined experimentally, they could not readily be expressed in mathematical form and the solution by both methods was graphical. Whilst by the Dynamic Programming method the solution is obtained by direct reading off the fuel consumption curve, the Lagrange Multiplier method involves graphical differentiation. The latter would, therefore, be expected to be the less accurate of the two.

In fact, the results obtained by the two methods for the minimum possible fleet fuel consumption differed by about 1 per cent. The actual fleet fuel consumption was found to be about 10 per cent higher than the theoretical minimum.

Owing to certain implicit assumptions in the calculation of the optimum consumption it was known that the whole of the difference was not in practice realizable. Another part of the difference was attributable to a few practices, the quantitative effects of which were evaluated in the course of the track tests. Appropriate steps were taken to obtain a saving in fuel consumption by having these practices corrected. It was then concluded that, by and large, driving methods in use were not unduly wasteful of fuel and that there was little scope for further savings in this direction.

The way in which the theoretical solution was obtained also enabled predictions to be made of the effect on fuel consumption of such factors as a change in the scheduled running speed of the fleet or in the average frequency of stopping. The close correspondence between actual and theoretical fuel consumptions lent validity to such predictions.

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STATEMENT OF PROBLEM

A vehicle makes a journey between two terminal points P_0 and P_n and has to do it in a scheduled time T. Between the termini it makes a series of stops, $P_1, P_2 \ldots, P_{n-1}$, which divide the journey into short sectors $(P_0P_1), (P_1P_2), (P_2P_3), \ldots, (P_{n-1}P_n)$ of length $d_1, d_2, \ldots, d_n$.

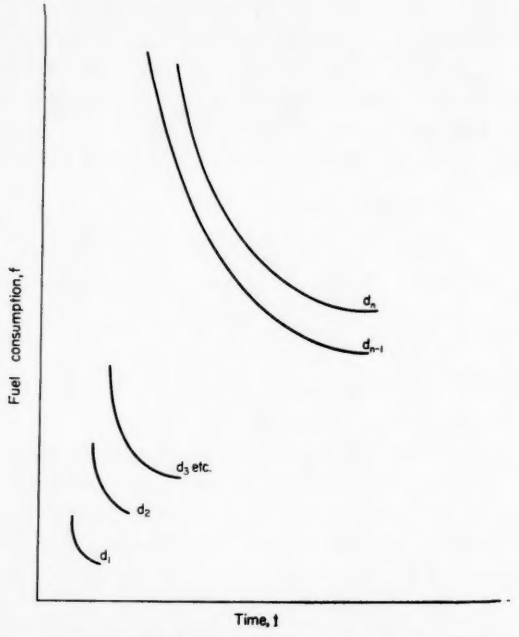

FIG. 1. *Minimal inter-stop fuel consumption as a function of time and distance.*

The minimal fuel consumption f of the vehicle between two adjacent stops depends on the time taken and may be represented by a function

$$f = \phi(d, t) \text{ where } \begin{aligned} f &= \text{inter-stop fuel consumption (minimal)}, \\ d &= \text{inter-stop distance}, \\ t &= \text{inter-stop time taken}. \end{aligned} \qquad (1)$$

For a given inter-stop section, i.e. constant d, the function is of the form shown in Figure 1.

For a given distance, minimal fuel consumption is a unique and monotonic decreasing function with t within certain limits of t. Fuel consumption may be assumed to decrease with increasing journey time over the whole range of conditions met with in practice, although the fuel consumption curve may rise again outside the practical range. In other words, very short journey times are expensive in fuel and involve "flogging" the vehicle, but if a longer time is

taken over the journey, fuel consumption is less. The upper limit of time is not of practical importance and the lower limit would not generally be reached since it implies that the journey is too tightly scheduled.

In the course of a complete journey a vehicle covers a series of such inter-stop sections of various lengths. The total fuel consumption is, of course, the sum of the individual inter-stop consumptions. The total time taken for a whole journey must conform to a pre-arranged schedule.

The problem is to find the minimum fuel consumption which can be obtained for a given total scheduled time. In other words, it is to find the optimum way of distributing the total scheduled time amongst the small inter-stop sections so as to minimize total fuel consumption.

Expressed mathematically, we wish to minimize the function

$$F = f_1 + f_2 + f_3 \ldots + f_n, \tag{2}$$

$$= \phi(d_1, t_1) + \phi(d_2, t_2) + \phi(d_3, t_3) \ldots + \phi(d_n, t_n), \tag{3}$$

for given values of $d_1, d_2, d_3, \ldots, d_n$ and subject to the condition

$$t_1 + t_2 + t_3 \ldots + t_n \Rightarrow T = \text{constant}. \tag{4}$$

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METHOD USING PRINCIPLES OF DYNAMIC PROGRAMMING

(1) *Application of Principle of Optimality*

Principle of Optimality*: "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

The application of the principle is as follows. Suppose that a vehicle performs a complete journey $(P_0 P_n)$ consisting of n sections, lengths $d_1, d_2, \ldots, d_n$, and takes times $t_1, t_2, \ldots, t_n$ such that the total fuel consumed F is optimal. If, in making this journey, it arrives at an intermediate point P_r at time T_r, then it follows from the Principle of Optimality that the way in which it has covered each section of the journey must be an optimum one for making the journey $P_0 P_r$ in time T_r.

(2) *Formulation of Dynamic Programming Solution*

The Dynamic Programming solution may be formulated in the usual mathematical notation as follows:

Let $g_r(T)$ be defined as the fuel usage under an optimum policy for covering r stages in a required time T, then

$$g_1(T) = \phi(d_1, T),$$

$$g_2(T) = \min_{t \leq T} [\phi(d_2, t) + g_1(T - t)],$$

$$g_n(T) = \min_{t \leq T} [\phi(d_n, t) + g_{n-1}(T - t)].$$

* R. BELLMAN. *Dynamic Programming*, p. 83.

(3) *Method of Solution*

The method used is to start with a single section and to add in the other sections one by one. At each stage a variety of conditions for the new section is tried and the optimum one chosen. In practice the inter-stop distances are grouped in the form of a frequency distribution.

(4) *First Section* (P_0P_1), *Length* d

The relationship between fuel consumption f and time t is given immediately by substituting distance d_1 in equation (1) illustrated in Figure 1.

(5) *First Two Sections* (P_0P_2), *Length* d_1+d_2

For the first two sections combined adopt any value of time, say T_2, and take

$$t_2 = T_2 - t_1. \tag{5}$$

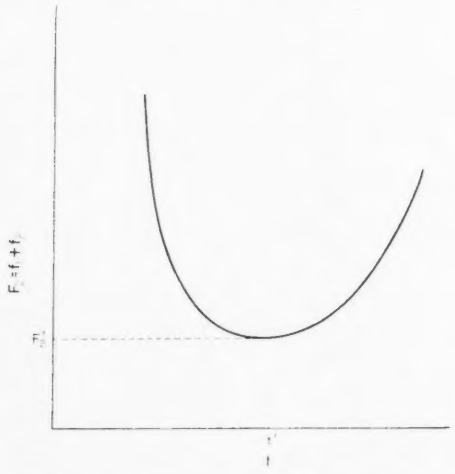

FIG. 2. *Fuel consumption* F_2 *over section* (P_0P_2) *for a given value of* T_2.

Various values of t_1 are now selected which automatically have corresponding values of t_2 according to equation 5. For each pair of values of t_1 and t_2, f_1 and f_2 can be calculated by substituting d_1, t_1 and d_2, t_2 in equation 1. When (f_1+f_2) is plotted against t_1 for the given value of T_2, a curve of the form given in Figure 2 is obtained.

The required optimal point for covering the section (P_0P_2) in time T_2 is given by (F_2', t_1').

This procedure is now repeated for different values of T_2 between its possible limits. From the optimal points applying to various values of T_2, a curve is plotted of F_2 against T_2 which will be of the form given in Figure 3. (The basic steps in the process are as set out in full above, but in practice it is possible

to simplify the computation by taking all possible values of t_1 and t_2 for the distances d_1 and d_2 and plotting $F_2 = f_1 + f_2$ against $T_2 = t_1 + t_2$. The envelope of all these points will then be the required curve of Figure 3.)

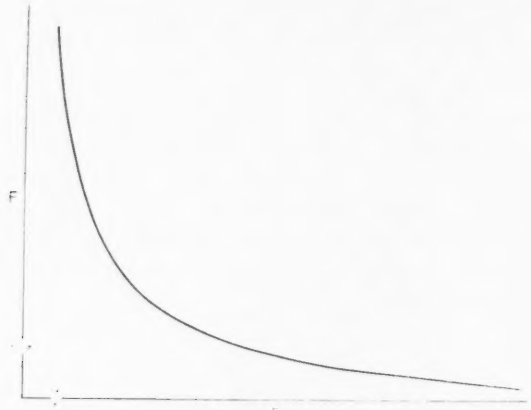

FIG. 3. *Relationship between optimal fuel consumption and scheduled time.*

(6) *First Three Sections* ($P_0 P_3$), *Length* $d_1 + d_2 + d_3$

For the first three sections combined, adopt any value T_3 within its possible limits and take

$$t_3 = T_3 - T_2. \tag{6}$$

By selecting corresponding pairs of T_2 and t_3 which satisfy equation 6, it is possible to draw curves similar to Figure 2 which apply to the values of T_3 chosen.

The optimum point of each curve is read off and hence a new curve of F_3 against T_3 can be plotted which will be of the same form as Figure 3.

(7) *Whole Journey* $P_0 P_n$

Further sections are added in as above until the complete journey has been built up and the required curve of F against T of similar shape to Figure 3 obtained.

From this curve the minimum possible fuel consumption for the whole journey can be read off for any value of scheduled time. Values of t_1, t_2, t_3, etc., which are required to specify the optimum method of driving can be obtained by going back over the intermediate working.

METHOD USING LAGRANGE'S MULTIPLIERS

Repeating the mathematical formulation of the problem given in equations (2), (3) and (4), we wish to minimize the total fuel consumption F where F is a

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function of the times taken to cover individual sections of journey, thus

$$F = f_1 + f_2 + f_3 \ldots + f_n, \tag{2}$$

$$= \phi_1(t_1) + \phi_2(t_2) + \phi_3(t_3) \ldots + \phi_n(t_n), \tag{3}$$

subject to a condition that the whole journey must be covered in total time T, so that

$$t_1 + t_2 + t_3 \ldots + t_n = T = \text{const.} \tag{4}$$

$\phi_r(t_r)$, $r = 1, 2, \ldots, n$, is a convenient notation for $\phi(d_r, t_r)$.

Lagrange's method of multipliers provides the solution to this kind of problem if we regard equation (3) as the function to be optimized and equation (4) as an equation of constraint.

Outlining the solution, we write down

$$\frac{\partial\{\phi_1(t_1)\}}{\partial t_1} dt_1 + \frac{\partial\{\phi_2(t_2)\}}{\partial t_2} dt_2 + \ldots + \frac{\partial\{\phi_n(t_n)\}}{\partial t_n} = 0 \tag{7}$$

and

$$\lambda dt_1 + \lambda dt_2 + \lambda dt_3 \ldots + \lambda dt_n = 0 \tag{8}$$

where λ is a Lagrangian multiplier.

Hence

$$\begin{aligned} \frac{\partial\{\phi_1(t_1)\}}{\partial t_1} + \lambda &= 0, \\ \frac{\partial\{\phi_2(t_2)\}}{\partial t_2} + \lambda &= 0, \\ \frac{\partial\{\phi_n(t_n)\}}{\partial t_n} + \lambda &= 0. \end{aligned} \tag{9}$$

So that the required condition for optimum F is that

$$\frac{\partial\{\phi_1(t_1)\}}{\partial t_1} = \frac{\partial\{\phi_2(t_2)\}}{\partial t_2} = \ldots = \frac{\partial\{\phi_n(t_n)\}}{\partial t_n} \tag{10}$$

In practice the solution has to be obtained graphically as the equation $f = \phi(t)$ cannot readily be expressed in mathematical form.

For the graphical solution a new series of curves of $\partial\{\phi(t)\}/\partial t$ is compiled by reading slopes off Figure 1. These curves are of the form shown in Figure 4.

The required condition for minimum F is obtained from Figure 4 by choosing a given value of the ordinate and reading off values of t corresponding to the various sections of route of lengths d_1, d_2, d_3, etc. These values of t are then used in conjunction with Figure 1 to calculate total F and draw the required curve of F against T applying to the whole journey which will be of similar shape to Figure 3.

This method is in a sense more interesting than the solution by the principle of optimality in that it gives an insight into the practical situation. We know that the harder a vehicle is pushed, so its fuel consumption increases even more rapidly. This fact is expressed by the shape of the curves in Figure 1. If a

vehicle is driven hard over one section of route so that one slope on Figure 1 is steep, but easily over the next section so that another slope on Figure 1 is flatter, then it is possible to save fuel but take no more time by equalizing the

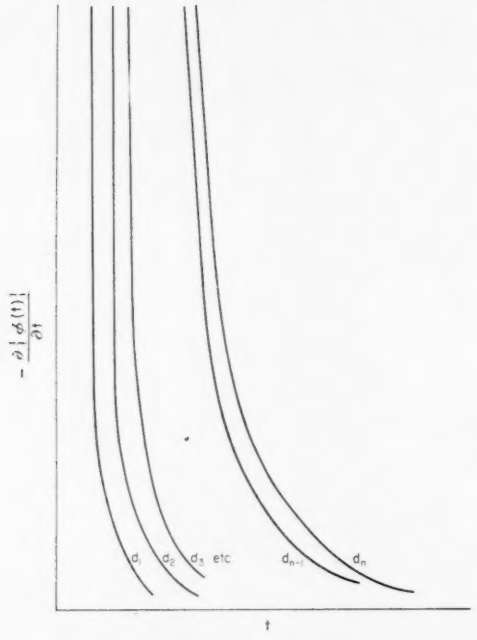

FIG. 4. *Curves of* $\frac{\partial\{\phi(t)\}}{\partial t}$ *against t (values of ordinate negative).*

two slopes and driving in a more moderate manner over both sections. In other words, $\partial\{\phi(t)\}/\partial t$ may be regarded as an index of "hardness of driving", and the minimal fuel consumption is obtained by driving with equal "hardness" over all sections so that the $\partial\{\phi(t)\}/\partial t$'s are equal.

ACKNOWLEDGEMENT

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Operational Research and Advertising: Theories of Response

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INTRODUCTION

IN THIS paper two of the authors (B. Benjamin and J. Maitland) provide further validation of a previously developed analogy between advertising and physiological stimulus and response. This is a simple analogy applicable only to this relation between *total* advertising and *total* response. To deal with this distribution of response in time a more complex epidemiological model is then developed and tested.

The third author (W. P. Jolly—a physicist) investigating the possibility of electronic simulation of advertising response, developed independently a simple exponential expression for coupon response which appeared to agree with experimental results and the physiological model.

TOTAL RESPONSE

The following data were made available relating to a press advertising campaign for a particular product (in this statement "cost" is related to expenditure and "results" to sales, in each case by a scale factor to conceal absolute values).

TABLE 1

Paper	Date	Cost	Results
Daily newspaper A (full page)	9 Sept.	3,250	12,373
Daily newspaper B (full page)	27 Aug.	700	3,657
	10 Sept.	700	3,146
Daily newspaper C (full page)	27 Aug.	2,352	11,457
	20 Oct.	2,352	11,140
Daily newspaper D (full page)	16 Sept.	5,000	18,745
Daily newspaper E	22 Oct.	2,288	5,232
Weekly F (full page)	28 Aug.	1,600	5,831
	15 Oct.	1,600	4,046
Weekly G (full page)	20 Aug.	4,400	11,276
	8 Oct.	4,400	10,962
Weekly N (full page)	16 Sept.	1,760	5,676

The data were first grouped so as to distinguish spells of advertising separated by intervals of time of sufficient length to justify this grouping. It must be emphasized that this arrangement of the data, although a matter of subjective

judgement, preceded the statistical analysis and was not made to fit that analysis. The grouped data are shown in Table 2:

TABLE 2

Date	Cost	Results
20 August	4,400	11,276
27, 28 ,,	4,652	20,945
9, 10 September	3,950	15,519
16 ,,	6,760	24,421
8 October	4,400	10,962
15 ,,	1,600	4,046
20, 22 ,,	4,640	16,372

These data were then examined for confirmation or rejection of the analogy of physiological response to stimulus suggested in a previous study[1] (Benjamin and Maitland, 1958) and expressed by the relationship

$$R = a \log_e A + b$$

where R is the response to, and A the amount of advertising, a and b being constants.

Figure 1 shows the plot of R against A on semilog scale (in each case in units of 1000). Except for two points (marked C) there is an approximation to a straight line. The line DE has the equation

$$R = 13{\cdot}991 \log_e A - 2{\cdot}841$$

and is a good least squares fit to all the points except those marked C.

The observed and calculated values of response are given in Table 3 (unit 1000)

TABLE 3

Observed	Calculated	Difference
4·05	3·73	·32
15·52	16·38	·86
16·37	18·64	2·27
20·95	18·66	2·29
24·42	23·90	·52

The measurement of advertising in this example is by *cost* and the points marked C in *both* cases relate not only to the same journal but to a journal which is well known to be costly in the advertising trade. The deviation from the fitted line is thus to be expected. It will be noted that the two spells for this journal give almost the same responses.

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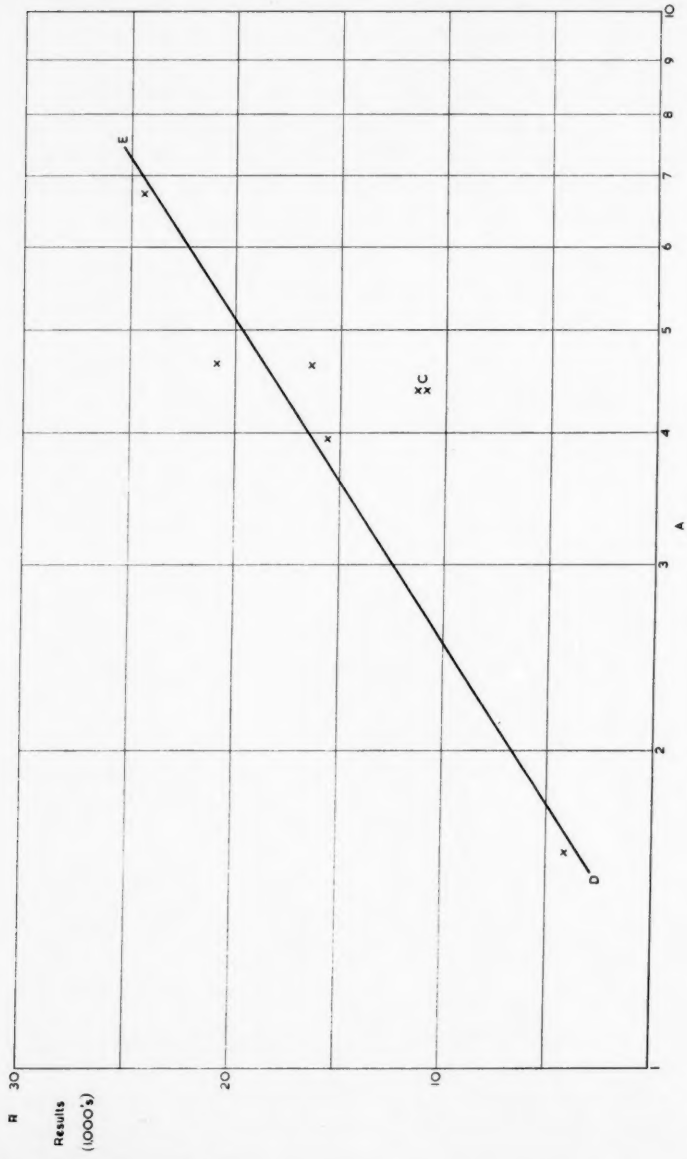

FIG. 1.

Time Response – A New Analogy

Only the total response to a given total amount of advertising has previously been considered. This response (and indeed, the stimulation of advertising) is, however, distributed in space and time. Figure 2 shows a typical build-up and

decay of sales from an advertisement in a journal. The bell-shaped type of curve is very similar to that for an epidemic of (say) measles or poliomyelitis with cases of infection occurring at fixed intervals of time. The epidemic analogy does indeed have an immediate appeal on several counts:

(i) Advertising is concerned with the spread of an idea ("X adds brightness", "Y is good for you", "Buy Z now") and it is desired that the idea shall take root in a receptive mind, as an infecting organism invades a susceptible body.

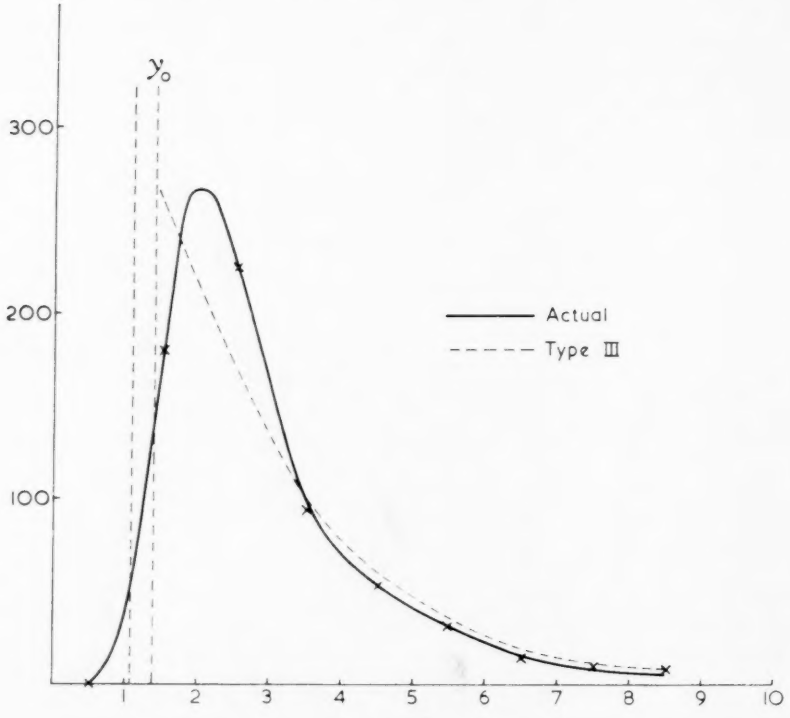

FIG. 2.

(ii) Some minds are not receptive to the idea, as some bodies escape disease although exposed to infection.

(iii) It takes time for the idea to take root and lead to action just as in bacteriological and viral infection there is usually an "incubation period" between infection and appearance of symptoms.

(iv) The ideas in an advertisement need not be simultaneously communicated to all persons in the community. They may be passed on from person to person by word of mouth just as "coughs and sneezes spread diseases". Indeed just as dysentery may be spread by carriers who harbour the

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infecting organism within their digestive systems without displaying the overt symptoms of disease, so advertising ideas may be spread by carriers who influence others but do not themselves purchase or take action.

Immunity and Susceptibility

In applying the epidemiological analogy to the spread of ideas the concepts normally employed need to be adapted to the conditions known to apply in advertising. When, for example, a person is attacked by an infecting organism, he suffers certain immunological reactions which leave him protected against further attack for a period of time. This may be a very short interval, as for the common cold, or may be virtually lifelong, as for measles.

This analogy is not an exact one. Where the reception of an advertisement prompts a relatively large capital outlay as for example for a motor car, exhaustion of personal financial reserves may operate as a temporary restraint against any further transactions of the same kind. An exchange transaction would involve little expenditure but would, of course, constitute a reaction to a new idea. Where the article involves small expenditure, for example, a packet of cigarettes or a can of beer, the subject can be (and probably needs to be) constantly reassured that he is indulging in the right brand, and here there is a continual process of reinfection. It seems desirable therefore to consider first expenditure of a once-for-all type and to regard immunity as operating for the normal lifetime of the particular commodity involved, at the end of which the subject has to be "reinfected" before replicating the purchase. Repetitive expenditure on small items has then to be considered as a special case which has not been developed in this paper.

Just as the concept of immunity is different for advertising so too is the concept of susceptibility. To an extent perhaps even greater than in communicable diseases there is considerable variation in susceptibility between one subject and another. Clearly those with empty purses or no money to spare are not immediately susceptible. Their financial stringency lends them temporary immunity. A parallel for this exists in epidemiology; it has been suggested that for certain diseases, especially measles, a condition of "latent immunity" of a transient character can be developed. Apart from shortage of money, effective susceptibility could be destroyed by a failure to make the advertised goods available in the right place at the right time. But these are purely physical restraints. Over and above this there is undoubtedly a wide variation in values from person to person. In developing the epidemiological model for advertising, therefore, one would expect to use the stochastic approach that has been developed by Bartlett[2] and Bailey[3]. It is however proposed for the present to adopt a purely *deterministic* approach, viz. that for a given set of conditions there will be an "attack" rate yielding a definite number of infected cases from a given number of susceptibles.

Incubation

A finite interval of time normally separates the reception of an advertisement and the act of purchase. The value content of the advertisement has to be appreciated and compared with other known values. For any given product and for any set of commercial conditions this time lag in response may be plausibly regarded as a constant. In the deterministic treatment of the spread of infectious disease the "incubation" period between infection and reaction is in fact treated as a constant though in practice there is some variation from individual to individual; the accepted incubation period is an average of observed values, some longer, some shorter.

The Vector

In the transmission of infectious disease an important part is played by the agent or vector which transfers the causative organism from the infected to the uninfected person. The vector may be droplets in the atmosphere as in many of the common fevers of childhood such as measles or chickenpox; it may be an insect as in malaria or some outbreaks of enteritis (the housefly); it may be an animal (the rat) as in leptospiral jaundice, it may be food as in trichiniasis; and it may be a human carrier as in some contagious diseases. Clearly the facility and speed of transmission of disease is dependent upon the type of vector involved. This is true also of advertising: a number of different media are employed (newspapers, hoardings, television, etc.) and the choice of the appropriate medium is important.

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A SIMPLE CASE

With these various factors in mind we may consider a simple case of advertising and examine the degree of analogy with communicable disease. Suppose we consider the single insertion of an advertisement in a journal with a circulation of say 100,000. Let us assume that this advertisement will be read simultaneously by 100,000 persons—a mass infection at a single point in time. At the end of the "incubation period" there will be a sudden jump to a modal response $y_0 = 100{,}000\, p_1$ where p_1 is the direct attack rate. In addition there will be communication to non-readers by the original readers and if this is mainly to intending or actual purchasers by those previously infected there will be at the end of a further incubation period another crop of purchasers $y_1 = y_0 \cdot p_2$ where p_2 is the secondary attack rate. Similarly these purchasers will influence others and at the end of a further incubation interval there will be a response $y_2 = y_1 p_2$. In this way there will be a logarithmic decline such that

$$\log y_{r+2} - \log y_{r+1} = \log p_2$$

One would expect a picture not unlike that represented in Figure 3.

Deviations from the Simple Case

It has to be borne in mind that the incubation period is not an interval of time which is the same for all persons who suffer infection. The times are distributed about the average which has been assumed to enter into the above

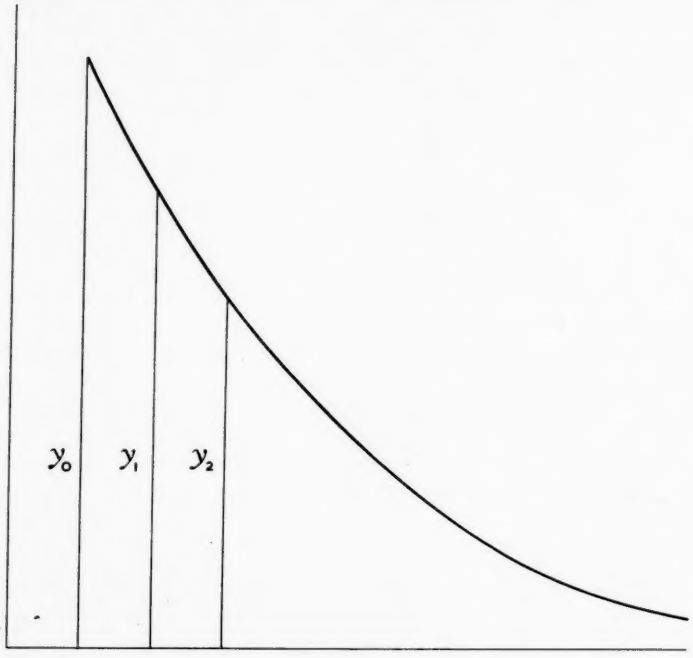

FIG. 3.

calculations and the rise to the *mode* will not therefore be instantaneous. For the same reason the secondary, tertiary, etc., generations of infection do not occur instantaneously. They are spread over time and the spread is greater for later generations.

PRACTICAL APPLICATION OF ANALOGY

The data in Table 4 represents the response in weekly sales following a single impulse advertisement in a journal. The data are those plotted in Figure 2. Examination of the moments of this distribution suggested the fit of a Pearson Type III curve. The curve has the equation

$$y = 24921 \exp[(-\cdot 5914x)] \left(1 + \frac{x}{\cdot 3091}\right)^{\cdot 1828}$$

where y = sales and x = time in weeks.

TABLE 4

Product X. Weekly journal
(Unit 100)

Week	Actual sales	Fitted type III curve
1	3	0
2	180	204
3	222	176
4	96	106
5	54	62
6	31	36
7	17	21
8	13	12
9	8	7

The fit is not particularly good; however, the main distortion occurs in the figures for the second and third weeks while the remainder convey a satisfactory impression of the general distribution of sales. A further example is provided by the data in Table 5:

TABLE 5

Advertisement in a weekly journal (with reference to it in a T.V. programme)

Week	⅛ (A) 12 Sept.	⅛ (B) 19 Sept.	⅛ (C) 26 Sept.	½ (D) 3 Oct.	(1) (2) Scaled down to one-third	⅛ (E) 10 Oct.	⅛ (F) 17 Oct.	⅛ (G) 24 Oct.	⅛ (H) 31 Oct.	Total Replies scaled to ⅛ pages
1	5									5
2	412	2								414
3	287	159	2							448
4	61	236	220	6	2					519
5	12	26	115	614	204	1				358
6	4	6	19	236	79	121	3			232
7	4	3	6	47	16	154	88	1		272
8	—	—	8	17	6	24	133	91	3	265
9	—	—	4	7	2	5	22	26	71	130
10	—	—	1	7	2	1	5	11	77	97
	785	432	383	934	311	307	251	129	151	

The figures in Table 5 are of interest in two ways:

1. The time distributions for each response are generally similar in shape though some are rather less skew than others. As shown in Table 6 the figures

in the first column, for example, are a reasonable fit to a Type III curve given by the equation

$$y = 1282\,[\exp(-1{\cdot}244x)]\left(1-\frac{x}{{\cdot}08504}\right)^{-{\cdot}1058}$$

TABLE 6

Week	Observed	Calculated
1	5	0
2	412	441
3	287	254
4	61	66
5	12	17
6	4	5
7	4	·2

2. Repeated weekly advertisements on the same scale produce for this product a declining specific response to each successive impulse (780 replies for the first, 151 replies to the eighth advertisement). The overall time distribution of replies to evenly spaced advertisements which is shown by the last column of Table 5 manifests a rising trend (after the initial surge) and a subsequent decline and is much smoother than the distribution specific to any one advertisement.

THE CHOICE OF CURVE

The data thus suggest a response distribution over time with a very sharp rise to a maximum and a subsequent gradual decline. The initial rise is not vertical as would be expected from the simple model in Figure 3. One reason for this has already been suggested, viz. the fact that the "incubation period" has a distribution and not a single value for all respondents.

A second reason is that the impact of the advertising "infection" will not be at exactly the same point of time for every member of the population affected. In the case of journal advertising, for example, a particular page will be read by different readers at different times—technical journals are sometimes not perused until several weeks, even months after the date of issue. For purposes of economy in manpower, however, mail coupon distribution may be spread over several days. Poster advertising, which requires the reader to come to the advertisement rather than the advertisement coming to the reader is likely to involve a long interval of time between the infection of the first victim and that of the last. Television of the single-shot type is the nearest approach to simultaneous mass infection.

These departures from simplicity suggest that an exact fit for an experiment may produce a mathematical equation of little use in another. It would seem

justifiable to find a curve of more general utility even if an element of approximation is introduced. Moreover, it has to be borne in mind that the fitting of Type III distributions is not likely to commend itself to advertising personnel as a simple or rapid procedure. For practical purposes there seems to be no reason why any given curve should not be used so long as it constitutes an approximation to the response distribution sufficiently close to yield correct decisions about the disposition of the product in retail centres. It could reasonably be assumed, for example, that the rise to the maximum response r_0 is exponential, $r = r_0[1-\exp(-k_1 t)]$ where t is time and that the subsequent decline is also exponential $r = r_0 \exp(-k_2)(t-n)$ where n is the point of time at which the modal response is reached. Measuring time from the beginning of the week in which the advertisement appeared, n might be taken as $1\frac{1}{2}$ weeks and the initial rise may be assumed to commence at $t = 1$. For the case shown in Figure 2 we have:

$$r = 250\{1-\exp[-5{\cdot}08(t-1)]\} \text{ Starting at } t = 1, \quad (1)$$

$$r = 250\{\exp[-{\cdot}46(t-1{\cdot}5)]\} \text{ Starting at } t = 1{\cdot}5, \quad (2)$$

yielding the distribution in Table 7.

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TABLE 7

Week	Observed	Calculated
1	3	0
2	180	213
3	222	161
4	96	100
5	54	63
6	31	40
7	17	24
8	13	14
9	8	9

If for practical purposes an instantaneous peak is assumed so that only equation (2) need be considered, the cumulative total response at time t is of the form $R_0[1-\exp(-kt)]$ where R_0 is now the final total response. In the above example the initial peak would be taken at 315 to leave the curve unchanged so that k is still ·46 and $R_0 = 315/{\cdot}46$ or 685. In the above statement the response for the second week becomes 253 and the remaining values are unchanged. It is this form which is considered in the next section.

ELECTRONIC SIMULATION OF ADVERTISING RESPONSE

In an investigation of the possibility of electronic simulation to predict advertising response it was decided to start with the relatively simple field of coupon

response since the number of coupons returned from a particular published advertisement is known. Furthermore a brief consideration of the problem suggested two fairly obvious "laws" which were simple, plausible and amenable to simulation. These laws gave results in reasonable agreement with past campaigns and were compatible with the more general theory quoted above.

The first of two laws proposed is that if a single advertisement is published the cumulative total of coupons returned rises exponentially with time to its final value: $r = r_0[1 - \exp(-k_1)]$ where r is the number of replies that have been received at any time t after the publication of the advertisement, R_0 is the final total of the replies received, e is the base of natural logarithms and k_1 is a constant which depends on the item advertised and the periodical carrying the advertisement.

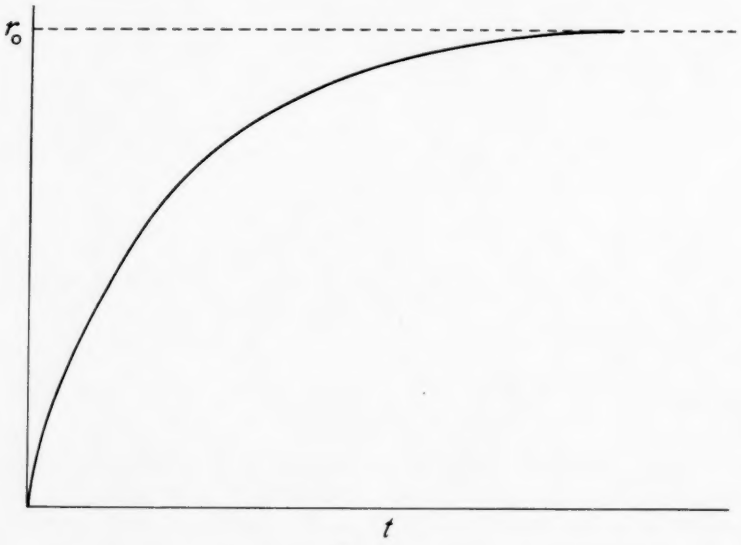

FIG. 4.

The curve relating r and time will thus have the form shown in Figure 4 and is in good agreement with the results available for advertisements of jewellery, garden equipment, wool, a travel brochure and for Service recruitment.

The second law proposed is that if the same advertisement is inserted regularly in a given periodical the total number of replies received to each advertisement will fall exponentially with time to some fixed steady value. This is an instance of the law of diminishing returns and the validity was confirmed in the limited number of campaigns for which suitable results were available.

Thus for repeated insertions of the same advertisement the replies received from each will follow the first law stated above but the value of r_0 for each

successive insertion will decrease so that

$$r_0 = R_0 + R_1 \exp(-k_2 t).$$

Where $R_0 + R_1$ is the total number of replies received to the first advertisement and R_0 the total number received to an advertisement inserted late in the campaign (R_0 is the steady value). The constant k_2 would be expected to vary with the subject advertised and the periodical. The curve of r_0 against time for regular insertions of the same advertisement is shown in Figure 5.

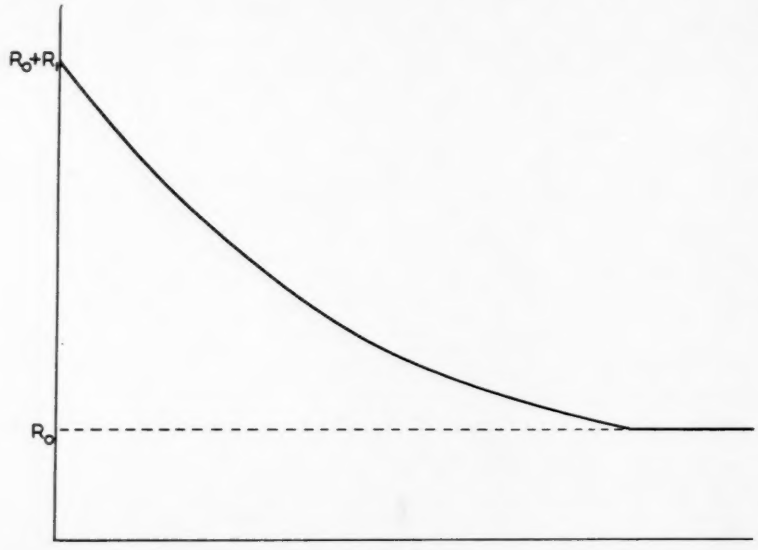

FIG. 5.

To these so-called fundamental laws is added a third empirical law based on the results of past campaigns. If one of the series of advertisements is omitted late in the campaign then there is a dip in the number of replies received but the response to the next insertion will show a peak suggesting that response is merely held back by the missing advertisement and postponed until the appearance of the next one. In the campaigns for which results were available only two cases of the omission of an advertisement occurred and these gave the "kink" suggested. It was of interest that all results showed the same "kink" over the December–January period, suggesting that Christmas has much the same effect as a missing advertisement.

Using the two fundamental and empirical laws it is possible to predict the shape of the curve showing the weekly (or monthly) replies received as a function of time when an advertisement is inserted regularly.

In Figure 6 the predicted curve is compared with the actual results of campaigns concerning the same product in three different periodicals. The responses have

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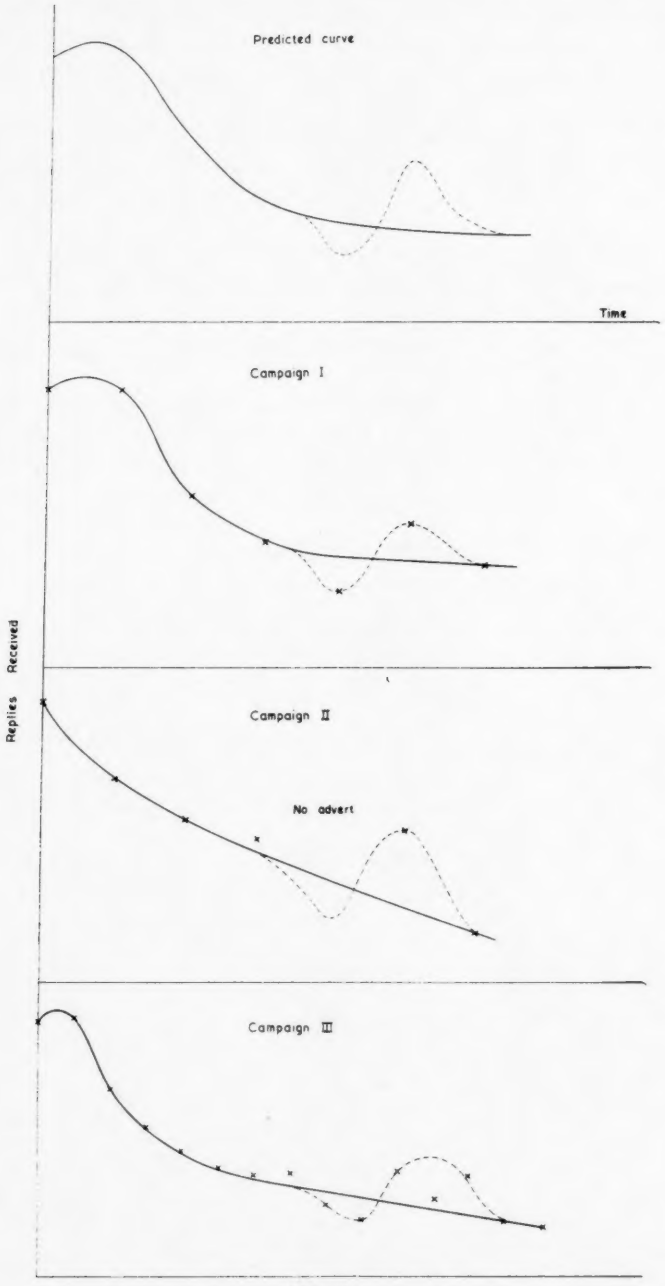
Predicted curve
Time
Campaign I
Campaign II
No advert
Campaign III
Replies Received

FIG. 6.

been reduced to the same scale in each case and the seven-month period, which includes Christmas, was the same in each case. The agreement between the curves is encouraging and suggests that this simple preliminary theory is worth pursuing.

Perhaps the most interesting practical feature of the theory is the information it promises to yield about the cost of advertising, a feature which was not considered in the formulation of the basic expressions used in predicting the response.

In any one of the campaigns shown in Figure 6 the points fell reasonably close to the predicted curve although the cost of the individual advertisements in the campaign varied by a factor of four or five. It thus appears that in many cases the cost of advertising could have been reduced without significantly affecting the response.

Following this simple theory an electronic analogue simulator has been designed and built for the prediction of the coupon response to be expected from any proposed press advertising campaign. An account of the simulator and its use in media planning will be published in due course.

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ACKNOWLEDGEMENTS

We wish to record our thanks for the interest and assistance shown to us during these investigations by The Board of Admiralty, The Air Council, Mr. Carl Brunning of S. C. Peacock & Co. Ltd., and Dr. George Macdonald, C.M.G., Professor of Tropical Hygiene, London School of Hygiene and Tropical Medicine (University of London).

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Operations Room Technique in Operational Research*

B. D. HANKIN

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(formerly of London Transport Executive)

INTRODUCTION

MANY operational researchers are called upon to study complex situations which are changing with time. It must be frankly admitted that they do not always find this very easy. The inherent complexity and change in the situations they are trying to measure make it extremely difficult to study the size and inter-relationship of all the various factors involved. This paper outlines a method which has been found useful in dealing with many situations of this type. It is an extension of the methods used for many years in the Services and elsewhere for the control of mobile forces.

ANALOGY WITH MILITARY AND CIVIL OPERATIONS ROOMS

Let us first consider what happens in a military or civil operations room. The photographs illustrate what some of these places look like, and they will no doubt be familiar to those who have been in the Services or have watched films on wartime subjects. It is suggested that in all cases the following features are common:

(a) The collection of *selected* information about a complex and changing situation.
(b) The transmission of information to a central point by a variety of communication methods.
(c) The display of the information in a form which makes it visually comprehensible to officers whose job it is to study it. (This usually involves some form of simple analysis or classification prior to display.)
(d) A study of the display by those on duty and the reaching of conclusions.
(e) Executive action if this is needed, i.e. control based on all the available sources of information. The display is probably not the only source but it makes an important contribution.

Consider now what a man engaged in operational research may be trying to do. He may, for example, be attempting to optimize or improve a situation which may vary from a relatively simple flow process to an extremely complex system whose component parts are distributed geographically in a manner which makes simultaneous research on the components difficult to arrange.

A common thread running through all these problems is that they are made up of a number of variables whose magnitude changes with time. Some of the

* Based on a paper given to the Operational Research Society on 26 February 1958.

variables are often inter-related and the whole pattern of events may be considerably influenced by chance. Some factors are more important than others, but the researcher may not know which they are. He is aiming to form a model but he cannot make much use of such a device until he has assembled a great deal of "real life" data and obtained a detailed practical understanding of the problem.

A common-sense way of understanding the problem is first to try to measure what is going on at present, even if at first it appears almost impossibly complex. It is suggested that to a first approximation this is exactly what the operations rooms and their staffs were doing in (a) to (d) above. Thus we have the basis of a method which can help us in operational research.

Definition of operations room technique. The following definition is suggested:

"The use of a Research Operations Room in order to *control* the collection of data, to carry out *primary analysis* and to *display* the results as an operation is taking place."

Four examples follow from a number of cases where this technique has been used successfully.

EXAMPLE 1 – STUDY OF COMMUNICATIONS AS A MEANS OF CONTROL OF MILITARY FORCES

In Germany in 1952/53 the Army Operational Research Group were studying the control of Divisional Forces. The ideal would have been to look at the problem in war, but peacetime manoeuvres were then on a large scale and were being made as realistic as possible by the Army. They thus formed a valuable opportunity to study the use made of the means of control and its relationship with the distribution of signals manpower and equipment. An important aspect concerned the study of the control of an Armoured Division.

To understand the need for the technique it is perhaps worth trying to visualize what an operationally deployed Armoured Division was like. At that time it consisted of approximately 17,000 men commanded by a Major-General, a Main Division Headquarters, a Rear Division Headquarters, an Armoured Brigade (tanks) and an Infantry Brigade (lorried infantry), and all the usual supporting arms and services. All the headquarters and troops concerned were highly mobile, concealing themselves in villages and woods. Vehicles had to be very carefully camouflaged and the whole of the Division was self-contained for eating, sleeping and fighting. When fairly concentrated it might have occupied various sections of a piece of territory 10 miles × 10 miles, i.e. 100 square miles in area. When moving (as it frequently did) it often became very much more stretched out than that. It will be appreciated that it was no simple problem to try to establish exactly what was happening in such a mobile and complex situation. The operational researchers had to be completely mobile and graft themselves on to the headquarters so that they were not in the way. If the study was to be complete they had to continue their measurements

FIG. 1. *Plotting convoy routes for the Royal Navy.* (*By courtesy of the Admiralty.*)

FIG. 2. *Royal Navy plotting room at Chatham.* (*By courtesy of the Admiralty.*)

FIG. 3. *Royal Air Force radar plotting room. (By courtesy of the Central Office of Information.)*

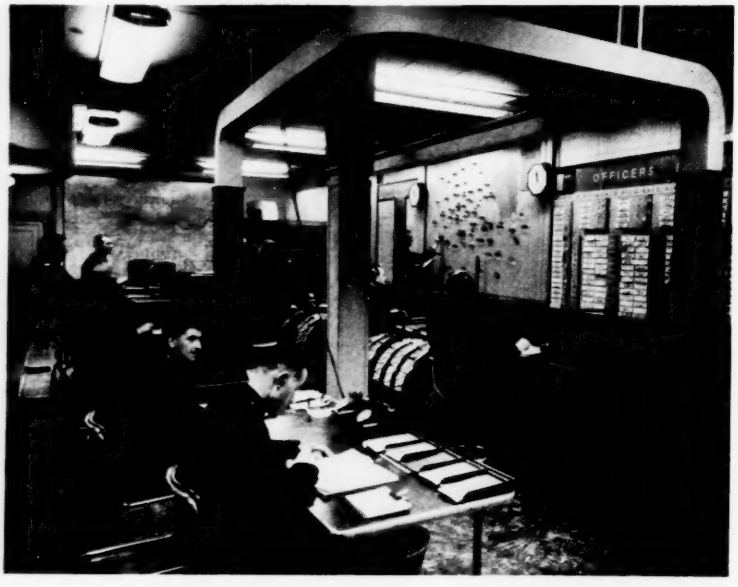

FIG. 4. *London Fire Brigade plotting room. (By courtesy of the Chief Officer and the L.C.C.)*

24 hours a day for perhaps a week, camouflage their vehicles, and live, eat and sleep as soldiers.

Operational Research had a caravan-type lorry as a headquarters, situated at Divisional Headquarters and moving with it. Other men and equipment were distributed at appropriate points consisting in all of about 24 men and 7 vehicles. The communications used in the Division at that time were radio-telephony, morse code by wireless, telephones, despatch rider and personal contact. The use made of these methods was measured as they varied with time. The results were sent at frequent intervals to the Research Headquarters, where they were charted to study their inter-relationship and dependence on the rapidly changing military situation. The value of this focal point for research was considerable and there is little doubt that the task could not have been satisfactorily accomplished without it. (Further information on this project was reported in *Operational Research Quarterly*, Volume 4, No. 4, December 1953, page 65.)

EXAMPLE 2 – STUDY OF TRAFFIC FLOW IN LONDON

Delays to buses at one of London's busiest intersections were being studied by London Transport's Operational Research Section. Shifts of inspectors with synchronized watches collected passing times of buses at selected points before and after the intersection in all directions. This process was kept up during the day for two weeks and observers' sheets were taken at frequent intervals to a plotting room, located very near the intersection. Large charts were prepared here on a time-scale which enabled a picture of what was going on hour by hour to be built up very soon after the event. Researchers were able to comprehend the situation in detail and investigate on the spot the considerable variability in delay which was being encountered. A similar method was useful in investigating the probable effect of a one-way street system which at one time was under consideration for alleviating congestion at the junction of Oxford Street and Regent Street. In this case both police and London Transport staff made use of the operations room. More recently, the Ministry of Transport's London Traffic Management Unit have used the technique to study traffic problems of the Motor Show and the Pink Zone.

EXAMPLE 3 – STUDY OF IRREGULARITY OF BUSES ON A BUS ROUTE

London Transport was studying the extent and contributory causes of irregular running on bus routes. A route approximately twelve miles long was selected, and the problem was to establish what was happening in all areas of the route simultaneously, allowing for the continual movement of the buses and changing traffic situations. This called for an Operations Room technique. The route was divided into six sections and staff were deployed with synchronized watches, covering the period of the day from about 8 a.m. to 7 p.m. The roadside staff

recorded the exact time that the buses passed them in either direction, together with their serial number and any other important factors affecting the flow of the buses. These sheets were collected up at intervals by supervisory staff, who telephoned the information to Research Headquarters using standard G.P.O. street call-boxes. The data were received and plotted on large charts approximately 12 ft long and 40 in. wide within one or two hours of the events. By the evening the chart for the day was virtually complete. This process was kept up for two weeks on the whole route and a further week concentrating on two shorter sections of the route to examine them in more detail. The chart was on a time-distance basis, and showed the detailed movement of every bus. It enabled the gradual formation of gaps to be clearly detected and traced back to their source. The follow-up of the causes of these gaps was carried out as the research progressed. Any unusual circumstances could be followed up immediately they were brought to light by the charts.

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EXAMPLE 4 – STUDY OF THE FLOW OF TRAINS ON TUBE RAILWAYS

In connection with studies on the passenger capacity of a tube railway, it was necessary to examine in detail the flow of trains down a thirteen-mile section of a railway containing twenty stations. Observers with synchronized watches were placed on each station to read the time of arrival and departure of trains and varied information related to signals to a synchronized accuracy of a second or two. This was done during each evening peak for a fortnight. After each shift the data were taken by train to a central plotting room, deliberately located near the offices of those who controlled that particular line. A large time-distance chart was plotted showing in detail the flow of individual trains. This was completed by about 2 p.m. the following day, when a conference was held with officials responsible for the line. It was possible to see very clearly exactly what was happening on the line as a whole, where the trains were being delayed and for how long, the interaction of one train on another, and the consequent effects on the flow as a whole. Bottle-neck stations could be pinpointed and valuable contributory factors and data obtained by secondary analysis.

LESSONS FROM EXPERIENCE

Experience of a number of these studies has led the author to conclude the following lessons for anybody wishing to use the method:

(a) There is a need for a detailed background knowledge of the problems. Pilot surveys are essential to build up the necessary knowledge of the situation and to test the efficiency of the communication, analysis and plotting systems.

(b) There is always a difficult decision as to what to measure with the limited manpower available. It is usually best to select a basic framework and

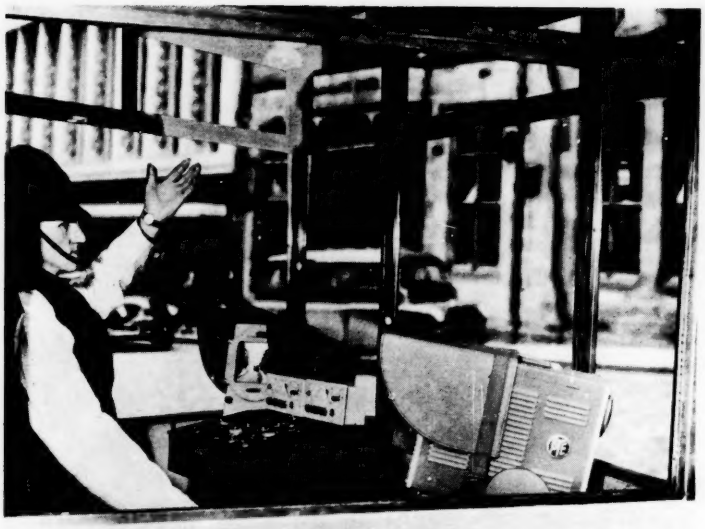

FIG. 5. *An unusual Operations Room. A Durham policeman using T.V. to assist in traffic control. (By courtesy of Messrs. Pye Ltd.)*

FIG. 6. *Operational Research Operations Room. Analysis and plotting staff participating in a study of bus delays at a major London intersection. (By courtesy of London Transport Executive.)*

RESEARCH ASSISTANT STUDYING BUS REGULARITY CHART
TRAFFIC CHART
SHIFT SUPERVISOR IN RESEARCH OFFICE
TWO CHART PLOTTERS
RESEARCH DATA ARRIVING BY PHONE

FIG. 7. *Operational Research Operations Room. Analysis and plotting staff participating in a study of regularity of bus running on a twelve-mile London route. (By courtesy of London Transport Executive.)*

FIG. 8. *Operational Research Operations Room. Analysis and plotting staff preparing a time-distance chart to show the flow of trains down a thirteen-mile section of Underground railway. (By courtesy of London Transport Executive.)*

use this during *the whole* of the research survey. Having carefully considered and made the decision on the basic framework of data, it pays to stick firmly to it and only deviate by small amounts if absolutely necessary as the work proceeds. (Standardized observations over a period of, say, a week are usually of much more value for drawing conclusions than an approach which changes day by day.) It is always possible to add to the basic framework by supplementary research.

(c) Great care is required in planning the primary analysis and display system. It should be simple, logical and capable of comprehension by all concerned, including both junior observers and analysers and management. All blank charts must be prepared and scaled beforehand, and if the problem demands it, should be sufficiently large to allow several people to work on them. This may be vital if the plotting process is to be carried out sufficiently quickly. Observers should be encouraged to call at the Operations Room to see the analysis and plotting of their data.

(d) Very careful briefing conferences are required for all the staff concerned.

(e) A small reserve of research manpower is essential in order to assist overloaded personnel in the early stages and to exploit or follow up interesting aspects which develop.

(f) The first day of a survey is always the most difficult. Staff get considerably more efficient after training and experience. This should be allowed for.

(g) Senior researchers must not be too deeply committed. They must be able to stand back to study the results and to co-ordinate the efforts of others.

(h) Unlike some of the Military Operations Rooms the records produced must be reasonably permanent to allow for secondary analysis. It is advisable to keep a detailed diary of special events.

(i) With suitable supervision it is surprising how much can be done in both data collection and simple analysis by borrowed staff who know nothing of research.

(j) There must be an attitude of mind spread from the top that the survey *must* succeed. Once the early trials have shown how to do the job, the final survey should be carefully planned and regarded like an operation of war that must succeed. An outlook of "let us see what we can do" does not lead to the best results.

ADVANTAGES AND DISADVANTAGES OF THE METHOD

Experience of a series of projects carried out in the manner described has suggested a number of advantages and disadvantages.

Disadvantages

(a) *Additional manpower.* Additional manpower is required during the survey to provide staff for the Operations Room to assist in the plotting and

analysis process. (It may be noted that the overall man hours needed to complete the project are not any greater by this method. In fact, because morale is invariably very high, an economy in *total* man hours may be achieved.)

(b) *Need for much preparation.* There is a need for much careful preparation and continuous hard work and supervision during the operation.

Advantages

(a) *Better control of research.* The Operations Room forms a focal point for information and control of research. Errors in research data or in the methods of collecting it can be quickly spotted and corrected. Small changes in the allocation of effort or the method of research can be made as the research proceeds, i.e. steersmanship based on feedback. Anything unusual can be followed up at once.

(b) *Speed in getting results.* The speed of production of an overall picture of the problem is very great. By the end of the survey the senior researchers have a very good idea of the problem, and do not need to wait several weeks for data to be analysed to give a summary of results.

(c) *High morale.* The morale of all analysis staff and observers is invariably high because they all feel closely involved. They are encouraged to come to the Operations Room and they feel that their efforts are needed urgently to build up the missing links in the jigsaw.

(d) *Discussion of results with management.* It is possible to communicate a broad picture of results to management as the survey proceeds. Selected members of management are invited to the Operations Room perhaps at daily intervals to see and discuss the results of the previous day's operations. This is extremely valuable to the operational researchers and of great interest to management. It is a useful step in the presentation and acceptance of results.

(e) *Better results from a given amount of measured data.* It is possible to get better results from a given amount of *measured* data. The well-known saying that "the results are as good as the data" is, in the opinion of the author, open to considerable misinterpretation. What you can get out of data depends on what you know of the problem and the situation of which it forms part. By living with the problem and by studying it by Operations Room technique the researchers have a far greater understanding of the problem and can make much more of the measured data. They are helped to select aspects for secondary analysis or special follow-up and are better able to present their conclusions to management. Their measurements can also be supplemented with visual impressions, which are very useful. (Compare with a selection board trying to come to conclusions about a candidate with only the application forms in

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front of them. The addition of an interview enables those responsible to get a much broader view of the person and the feel of the situation, and assists them to ask the right questions.)

USE OF THE METHOD BY OTHER ORGANIZATIONS

Early in 1958 an approach was made to some of the leading operational research groups in the United Kingdom, U.S.A. and in Europe to try to find out if much use has been made of this method. There was considerable interest, but it was clear that with one or two minor exceptions, which in some respects were similar, very little use had then been made of the technique. It appeared that many organizations were then satisfied to collect their data and study it at leisure after the events. There were doubts as to the availability of manpower, and as to whether a sufficiently rapid analysis could be kept up to keep pace with the incoming data. It is believed that since 1958 some organizations are beginning to experiment on the lines described.

CONCLUSION

In the experience of the author the technique can be achieved quite economically if it is carefully organized and if it is regarded only as a broad backbone analysis of the problem. It provides a valuable primary analysis and is an excellent data source for further secondary analysis. Some of this more detailed analysis can usually be going on during the operation, but some will certainly remain to be tackled after the completion of the data collection. All the examples described above have used junior staff to analyse and display the data. The increasing potentialities of computers as an aid to rapid analysis suggests almost unlimited possibilities of variants of this method either as a research tool or possibly as an aid to Management Control.

Production Planning and Inventory Control on a Chemical Plant

P. M. BLUCK, P. G. SMITH and G. THACKRAY*
The Distillers Company Limited

In this paper formulae are derived to give the optimum run lengths on a plant producing two products alternately. The general case of n runs of one product to one run of the other is considered. The work is applied to a chemical plant and the difficulties of determining certain costs are discussed. It is estimated that appreciable savings can be made.

An empirical method of calculating buffer stocks is then described, together with a method of planning production before an annual shut-down. This part of the work does not lead to any substantial savings in cost, but reduces the danger of running out of stock.

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INTRODUCTION

THE study described in this paper was concerned with production planning and inventory control on a chemical plant that is one of a number on a large site. The plant can be operated independently of the others on the site, but all services (e.g. steam, maintenance facilities) are organized for the site as a whole. Two products, both liquids, are made alternately on this plant.

Object of the Investigation

The general objective was to plan production in such a way that the total costs would be minimized, subject to certain restrictions regarding running out of stock. It was found that the investigation could be divided into two virtually independent parts, namely the determination of the optimum length of a production run and the determination of a minimum stock level at which production (of the product) must be started.

Description of the Plant

The process, which is continuous, consists of a reaction between the appropriate raw materials in a steam-heated reactor, followed by distillation of the crude product. Starting up the plant or changing from one product to another takes about two days; but there is no limit to the possible length of a run on either product.

The current annual sales of the two products were 45,000 and 13,000 units respectively,† and the production rates were 300 and 200 units/day.

* Mr. P. M. Bluck is now with the Mullard Radio Valve Co. Ltd. and Mr. G. Thackray is at the Carrington Research Laboratories of the Shell Chemical Co. Ltd.

† Because of the confidential nature of the work the two products are denoted by the letters A and B, and the quantities have been coded. The convention of placing the figures for Product A before those of Product B is adopted throughout this paper.

News and Notes

News of Members

Changes of Appointment

Professor R. L. Carter—Previously with Illinois Institute of Technology, is now with the Rensselaer Polytechnic Institute, Troy, N.Y.

R. Chapman—Previously with Stewarts & Lloyds Ltd., is now with BISRA, London.

K. H. Dawson—Previously with the Atomic Energy Group, GEC, is now with AIC Ltd.

F. B. Graham—Previously with Matson Navigation Co., is now with Arthur D. Little Inc., Massachusetts.

B. D. Hankin—Previously with London Transport Executive, is now with the Ministry of Transport, London Management Unit.

K. F. Lane—Previously with Rio Tinto Management Service is now with The Carborundum Co., Niagara Falls.

B. O. Marshall, jr.—Previously with Arthur Andersen & Co., is now with the Radio Corporation of America, Camden, N.J.

R. G. Massey—Previously with BISRA is now with Richard Thomas & Baldwins Ltd., Newport.

S. A. Pai—Previously with Associated Electrical Industries, is now with the Steel Company of Wales Ltd., Port Talbot.

D. R. Read—Previously with The Distillers Co. Ltd., is now with Cadbury Brothers Ltd., Birmingham.

Dr. R. H. Rogers—Previously with the Birmingham College of Technology, is now with the Willesden Technical College.

R. J. Semmens—Previously with Steel Peech & Tozer is now with The Esso Refinery, Fawley.

P. F. Wade—Previously with the Aluminium Co. of Canada, is now with Price Waterhouse & Co., Montreal.

P. I. Welding—Previously with London Transport Executive is now with British Transport Commission, London.

New Full Members of the Society

M. G. Brazil—The Wallace Attwood Co., London.

Dr. S. Eilon—Imperial College, London.

J. G. Fergusson—National Coal Board, London.

B. J. Walby—The Distillers Co. Ltd., Epsom.

J. K. Wyatt—Glacier Metal Co. Ltd., Wembley.

Congratulations

We congratulate Dr. Herbert Holt of The Center for Advanced Management, New Canaan, Connecticut, who subscribed a paper to our last issue, on the award of the Eloy Alfaro Grand Cross and Diploma, which is awarded by The Eloy Alfaro International Foundation, in recognition of his many contributions to the common good, both in war and peace.

We are pleased to record the award of one-half of the 1959 Lanchester Prize to Mr. A. M. Lee, British European Airways, for his paper "Some Aspects of a Control and Communication System" which was published in the *Operational Research Quarterly*, December 1959, and we offer him our hearty congratulations.

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Midlands Operational Research Society

In December 1959 about 65 people attended a meeting which decided to form a local Operational Research Group. The first meeting was addressed by Mr. B. H. P. RIVETT whose subject was "The Allocation of Capital Investment".

The Society now has a membership of 68, which meets regularly during the year and provides an opportunity for listening to speakers and holding informal discussions. About one-third of the membership is actively engaged in operational research work and the other two-thirds are interested people from other professions. Meetings are generally held in Birmingham, although at least one meeting will take place in another Midland town.

Further details of the Society and its activities may be obtained from K. B. HALEY, Hon. Sec., Midlands Operational Research Society, The Institute for Engineering Production, The University of Birmingham, 16 Norfolk Road, Edgbaston, Birmingham, 15.

The North-Western Operational Research Group

The North-Western Operational Research Group was formally constituted in April 1959. It is based in Manchester and aims at covering the North-Western Area of England. A programme of monthly meetings has been arranged which will be held in the Manchester College of Science and Technology, unless otherwise stated. Inquiries on membership may be directed to the Hon. Secretary, D. S. STEWART, Ferodo Ltd., Chapel-en-le-Frith, Via Stockport, Cheshire. Information may also be had on the constitution, committee and activities.

Forthcoming meetings are:

1960

Tuesday, 20 December: C. S. Chedzey (A.I.C. Ltd.). "Queueing in Crown Post Offices".

1961

Tuesday, 17 January: B. H. P. Rivett (Arthur Andersen & Co.). "A critical survey of operational research in the light of the papers presented at the 1960 I.F.O.R.S. Conference."

Tuesday, 21 February: B. Griffiths (Shirley Institute). "Operational research in the Cotton Industry."

Friday, 24 March: J. R. S. Ives (British Oils & Cake Mills). "Production Planning and Control as applied to the manufacture of Animal Feeding Stuffs." This will be a joint meeting with the Industrial Group of the Manchester Statistical Society, and will take place at the Manchester Literary and Philosophical Society, 36 George Street, Manchester, at 6 p.m.

Tuesday, 18 April: W. D. Ewan (British Nylon Spinners). "Optimum cycle arrangement of interdependent machines."

AWARD OF THE PUBLISHING HOUSE WESTDEUTSCHER VERLAG, COLOGNE

For the promotion of science and research the Publishing House "Westdeutscher Verlag" at Cologne and Opladen announced in May 1959, on the occasion of the awarding of its first prize, a new prize for scientific treatises in the field of sociology, economics, business management and political science.

The prize is awarded in parts of DM 400 each for a treatise in the following four subjects: sociology, economics, business management and political science.

The Board of Trustees has made two topics optional in each field:

1. *Economics*
   (a) How far can the methods of activity or operational research be used for economic policy decisions?
   (b) The rent concept in economic theory—a systems analysis.

2. *Business Economics*
   (a) How far can the methods of activity analysis or operational research be used for entrepreneurial decisions?
   (b) Business price policy and modern theory of cost accounting.

3. *Sociology*
   (a) Small Group Research and its significance for Sociological Theory.
   (b) Sociological Problems of Education in Developed Industrial Societies.

4. *Political Science*
   (a) The theory of terror in a radical democracy.
   (b) The constitution of Federal Republic of Germany in practice in 1949 and 1959—a comparison.

Any German or foreign competitor can submit a treatise for this prize competition. Any treatise submitted must be an unpublished original which has at no time been submitted at any other occasion; it may be written in German, English or French.

The manuscripts have to be submitted not later than 30 June, 1961. Full details may be obtained from Westdeutscher Verlag, Opladen.

SOUTH-WESTERN OPERATIONAL RESEARCH DISCUSSION SOCIETY

Winter Programme 1960/61

1960

Thursday, 29 September: Dr. E. Koenigsberg (Touche, Ross, Bailey & Smart, U.S.A.). "Operational Research and Accounting."

Thursday, 20 October: J. Taylor (B.O.A.C.). "Some Aspects of Aircraft Maintenance and Spares Provisioning."

Tuesday, 22 November: Dr. W. Ross Ashby (Burden Neurological Institute). "New Developments in Cybernetics."

1961

Wednesday, 11 January: Dr. M. G. Simpson (Touche, Ross, Bailey & Smart, London). "Dynamic Programming."

Thursday, 9 February: J. W. Harling (Urwick Orr & Partners). "Operational Research in the Smaller Company."

Wednesday, 8 March: H. R. W. Watkins (Shell Mex & B.P. Ltd.). "Business Games in Business."

Tuesday, 11 April: Annual General Meeting.

All meetings will be held in Cardiff at 6.30 p.m. The September and February meetings will be held in the Park Hotel; subject to later confirmation, it is planned to hold the remaining meetings in the Welsh College of Advanced Technology.

Hon. Sec.: J. K. Friend, 5 Minehead Avenue, Sully, Penarth, Glamorgan.

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Existing storage capacity for the two finished products is 6700 units and 2000 units (enough to supply over six weeks' demand) and stocks have fluctuated from zero to 5500 units with Product A and zero to 2000 units with Product B.

The yearly pattern of production was to have about eight runs of Product A, each of 18–20 days, and about six runs of Product B, each of about 10 days. The run of Product B generally followed immediately after a run on Product A (apart from the two days needed to change over), giving "silent" periods of at least 10 days before the next run on Product A.

OPTIMUM RUN LENGTH

It was known that starting the plant for a production run and changing over from one product to another involved an appreciable cost, partly for steam,

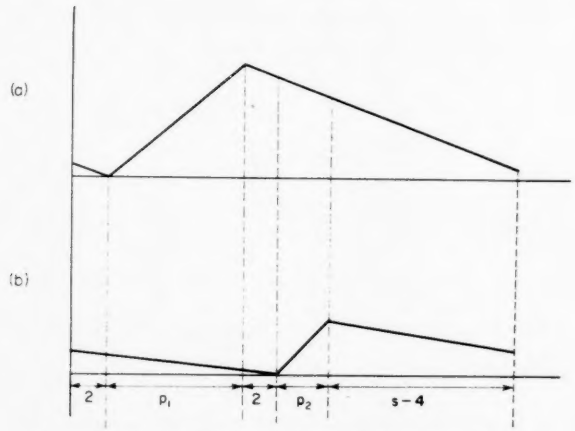

FIG. 1. *Stock levels on a two-product plant: normal cycle.*

but possibly also because of low efficiency before equilibrium conditions have been reached. For this reason longer runs led to a reduction in operating costs for a given production. Long runs, however, also lead to higher stocks and therefore higher inventory costs. The optimum run length is that which minimizes the sum of these two costs.

It is important to recognize two other types of cost, on which the solution to this problem does not depend. The first comprises all fixed costs (overheads) that are incurred at all times, whether the plant is on steam or not. On this plant these include charges for staff, depreciation, laboratory services, maintenance, steam for heating, and, subject to certain restrictions discussed later, labour. The second type of cost includes only those costs incurred during a run; these will be the same for all schedules meeting a given sales demand.

Derivation of Formulae

The normal cycle of operation is to start the plant and run on Product A for several days and then to change over to Product B for several days. After this the plant remains "silent" for a few days until the next cycle is required. This is illustrated in Figure 1 and formulae for determining the optimum cycle length are developed in the Appendix.

More complicated cycles of operation are possible and a scheme involving two runs of Product A for each run of Product B is illustrated in Figure 2.

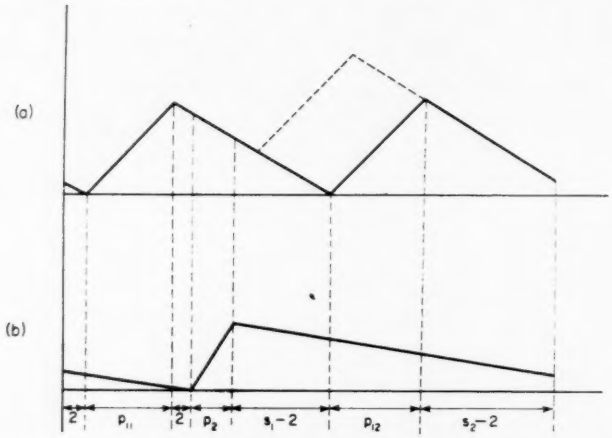

FIG. 2. *Stock levels on a two-product plant: split run cycle.*

A method of determining the optimum number of runs of each product in a cycle is given in the Appendix.

Inventory Costs

In most manufacturing concerns it is necessary to carry stocks of raw material and of finished products, mainly to act as a buffer for the variation in the difference between production and demand. Thus the cost of carrying stocks includes, therefore, the cost of providing storage space, insurance and depreciation costs, allowance for evaporation losses and deterioration and an allowance for the interest or investment value of the money tied up in stocks. The last of these is by far the most important in this problem.

This interest rate, however, is the figure most open to question. Many factors have to be taken into account in arriving at a realistic cost of capital charge, for it is affected by the general economic position at the time, sources of new capital available (for example, by bank or other loans or from shareholders), other competing demands for capital and the return which could be expected from such investment (for example, reduction of a loan, purchases of shares, construction of new production plant, etc.), the company policy

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towards liquid assets and the magnitude of the risks likely to be incurred. The problem is more difficult in solution for a large than for a small company and the right answer probably lies between 5 per cent and 35 per cent.

When valuing stocks for this purpose only the costs that would actually be saved if a small quantity of the material were not produced should be considered. These direct costs will not include any overheads, labour costs, laboratory charges, or maintenance charges. This leaves only the costs of services (steam, electricity and water), raw materials and other materials (e.g. catalysts).

For the purposes of this paper the overall inventory cost has been taken to be 20 per cent of the direct cost of manufacture. This figure, which includes all inventory costs, was felt to be appropriate in view of the economic conditions at the time when this work was carried out (1957).

Raw Material Stocks

It is necessary to consider the effect of changes in the cycle length on raw material stocks. This will depend on whether the raw materials are bought at a relatively steady rate (independently of production runs), in large consignments, or when required for production. In this plant the raw material common to both products is made on the site while the others are bought in large consignments. The usage by this plant is, however, small in comparison with other usages and sales of these materials, and consequently changes in cycle lengths and in buffer stocks will have no appreciable effect on the raw material stocks, once the plan has been in operation for a short time.

Numerical Values

Determination of the start-up cost C proved difficult. The cost of steam used during the start-up and change-over periods was calculated to be about £75 per cycle, when one run on each product is made per cycle. It appears that labour costs do not increase during the start-up and change-over periods, but this may not remain true if the optimum run lengths are, say, only one-half or one-third of those at present. The cost of other services appears to be negligible compared with that of steam.

It was suggested that there is a loss of yield at the beginning of a run, and in this context, the effect of such a loss is to increase the set-up cost by an amount equal to that of the material lost. Attempts were made to measure the loss by chemical analysis of waste gases and liquids and by a statistical analysis of operating records. This established that the total loss does not exceed 5 units per start-up and may be less than half this amount (i.e. total loss between 5 and 10 units per cycle). Since the manufacturing cost of Product A is £12/unit and of Product B is £9/unit, the corresponding values of C are £127·5 and £180.

The way in which the optimum cycle and run lengths vary with sales rate, production rate, set-up cost and inventory cost is demonstrated in Tables 1–3

and in Figure 3, for the case where one run is made on each product per cycle. In this figure the straight line shows the relation between annual inventory cost and cycle length. The three broken curves show the annual start-up costs

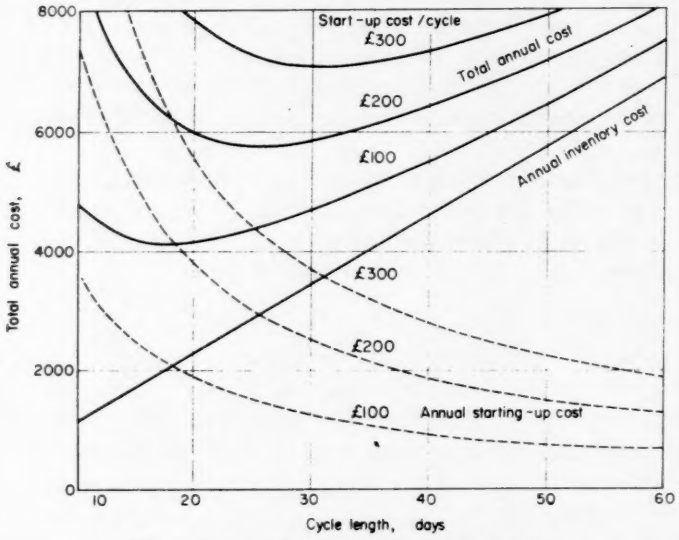

FIG. 3. *Relations between costs and cycle length.*

assuming the cost of start-up is £100, £200 or £300 per cycle. The top curves show how the optimum cycle length varies with start-up cost; they also demonstrate the penalties of working away from the optimum.

TABLE 1. VARIATION OF OPTIMUM RUN LENGTH WITH SET-UP COST

C (£)	75	127·5	180
p_1 (days)	6·4	8·3	9·8
p_2 (days)	2·8	3·6	4·3
r (days)	15·5	20·2	24·0
T (£)	3528	4600	5465

As can be seen from the equation derived previously, the run length is proportional to the square root of the two costs. The results in Table 1 show that *cycle* lengths should be well under one month. In this table production rates of 300 and 200 units/day, and sales rates of 45,000 and 13,000 units p.a. for A and B respectively have been assumed.

Because an accurate determination of the amount of material lost was not possible, a mean value of £150 has been taken for the start-up cost per cycle

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in constructing Tables 2 and 3. Fortunately, optimum values of the run length are not very sensitive to this value.

TABLE 2. VARIATION OF OPTIMUM RUN LENGTH WITH SALES RATE

Monthly sales (units)		Run length (days)		Cycle length r (days)	Cost T (£)
A	B	p_1	p_2		
3500	900	8·6	3·4	22·6	4860
3500	1100	8·4	4·0	22·1	4940
3500	1300	8·3	4·6	21·9	5013
3750	900	9·2	3·4	22·4	4911
3750	1100	9·0	3·9	21·9	4991
3750	1300	8·8	4·5	21·6	5063
4000	900	9·7	3·3	22·1	4947
4000	1100	9·6	3·9	21·8	5026
4000	1300	9·4	4·5	21·5	5097

Table 2 shows the way in which the optimum run lengths vary with sales rate, when production rates are 300 and 200 units/day. It can be seen that the cycle length varies only slightly. It was recommended, therefore, that a fixed cycle length, corresponding to the forecast annual sales rate, be used throughout the year, and the run lengths determined monthly as $p = rS/P$, using the forecast monthly sales rate S. (This point is discussed further in the section on operating the plan.)

TABLE 3. VARIATION OF OPTIMUM RUN LENGTH WITH PRODUCTION RATE

Daily production rate (units)		Run length (days)		Cycle length r (days)	Cost T (£)
A	B	p_1	p_2		
300	200	9·2	3·9	21·9	5002
300	225	9·2	3·5	21·8	5016
300	250	9·2	3·1	21·8	5028
350	200	7·6	3·8	21·1	5197
350	225	7·6	3·4	21·0	5211
350	250	7·6	3·0	21·0	5222
400	200	6·4	3·7	20·5	5339
400	225	6·4	3·3	20·5	5352
400	250	6·4	2·9	20·4	5362

The dependence of the cycle length on the production rate is demonstrated in Table 3 where the annual sales are taken to be 45,000 and 13,000 units respectively. As before, the optimum cycle length is comparatively insensitive to an increase in the production rate, but the run length varies rather more.

To determine if further savings can be made by having split runs, it is first necessary to determine the optimum value of n, the number of runs of A to every one of B (n may be a fraction). Now $C_1 = C_2 = 75$ (the yield loss of A is financially greater than that of B, but this is compensated by the greater steam usage on B when starting-up). For the production rates, 300 and 200 for A and B respectively, the equation gives $n_0 = 1{\cdot}81$ for which $n_0 = 2$ is a sufficiently good approximation.

The optimum cycle length is then 34 days, with 21 runs on A and 11 on B. The value of T is now £4806 instead of £5002. (The theoretical minimum with $n_0 = 1{\cdot}81$ gives a value of £4804 for T.)

MINIMUM STOCK LEVEL AND START-UP RULE

Statement of the Problem

It is company policy to have stock always available to meet an urgent demand for these products. Strictly, this policy implies an infinite stock; but a compromise solution to the minimum stock problem has been found that will give a very high level of protection against running out.

To illustrate the problem, consider the stocks of Product B at a time when the plant is either silent or producing Product A. When the stocks fall to some level Q it will be necessary to start producing Product B if the danger of running out of stock is to be avoided; the problem is to determine Q, which can be called "the basic danger level".

The following three assumptions are made:

(i) Stock levels are checked only once per day and the decision to produce or not to produce is made then.

(ii) It may be up to 48 hours after making the decision before the pure product can be taken off the plant.

(iii) Product made during the first few hours after take-off commences may not be ready for despatch that day. Thus the following is a possible time-table of events:

(1) Monday, 9 a.m.: Decision made *not* to produce. (Stocks greater than Q.)

(2) Tuesday, 9 a.m.: Decision made to produce. (Stocks less than Q.)

(3) Thursday, 9 a.m.: Pure product flows.

(4) Thursday, 5 p.m.: First "batch" tested and ready for despatch.

(5) Friday, 9 a.m.: 16 hours' make-ready for despatch (1 day's make is available for despatch on Friday).

To avoid running out of stock, the stock level at 9 a.m. on Monday must be sufficient to meet the demand on the four days Monday to Thursday and also any excess of demand over production on the following days.

Thus the main problem is to find the distribution of despatches over a period of four consecutive days and check that there is no excess of demand over production in the following days.

Analysis of Despatches

Originally it was intended to fit a distribution function to the daily despatch data, but a study of frequency histograms made it clear that this approach was not feasible because despatches of these products have a very clear pattern

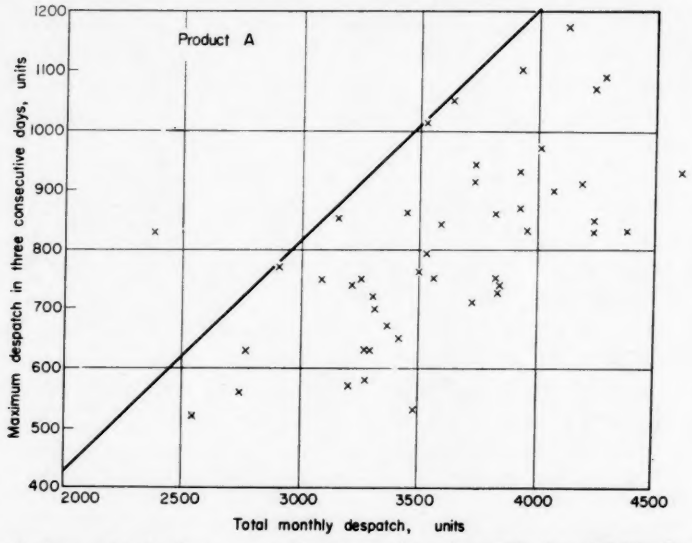

FIG. 4. *Despatches in three consecutive days and total monthly despatch (Product A)*

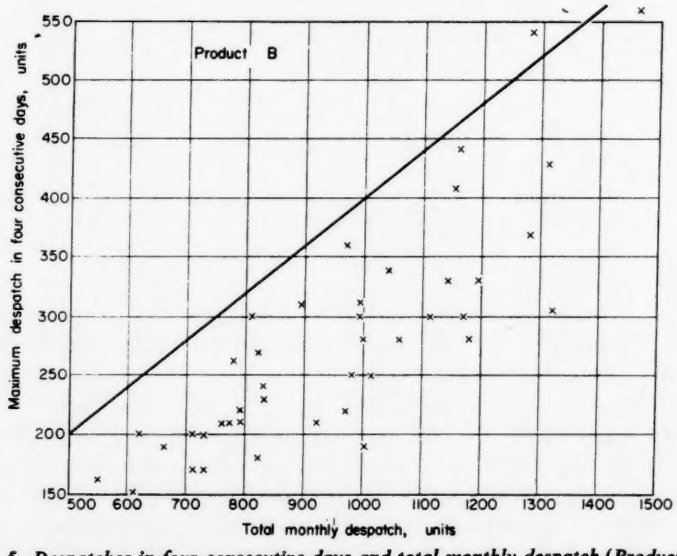

FIG. 5. *Despatches in four consecutive days and total monthly despatch (Product B)*

within each week. The ideal approach would therefore entail considering each day of the week separately, with some correction for trend within the month and the expected monthly sales, but there were insufficient data to make this analysis and another approach was necessary.

The method finally used was to study the relation between the maximum despatch on k ($k = 1, 2, ..., 9$) consecutive days in a month and the total despatch for that month. It is appreciated that this approach has some obvious shortcomings; but these are to a large extent outweighed by the straightforward solution obtained and the ease with which it can be implemented at the factory.

Graphs were plotted, for each value of k, showing maximum despatch on k consecutive days of the month (Y_k) against the total monthly despatch (D), over a period of 3 years. A straight line, $\hat{Y}_k = mD + c_k$, was drawn by eye such that $\hat{Y}_k > Y_k$ for almost all the observed values of D. Only obvious outliers were permitted to lie above the line; see Figures 4 and 5. This line gives an estimate of the maximum likely despatch on n consecutive days for any level of monthly sales. It was found that for Product B a suitable form of equation was given by putting $c_k = 0$. This enabled $\hat{Y}_k$ for Product B to be given in the form of a percentage of monthly sales; see Table 4.

Similar findings hold for Product A except that c_k is not zero, and hence the results cannot be expressed as a percentage; they are given in Table 5.

TABLE 4. MAXIMUM DESPATCH OF PRODUCT B IN k CONSECUTIVE DAYS, EXPRESSED AS A PERCENTAGE OF THE MONTHLY DESPATCH

k (Days)	Maximum despatch (%)
1	25
2	34
3	40
4	40
5	40
6	44
7	46
8	56
9	60

It will be seen that with Product B, 40 per cent of the monthly despatch could be sent out on four consecutive days, so that with a production rate of 200 units/day there is no danger of running out after the four days have elapsed if Q were taken as 0·4 D. This leads to a simple rule that Q for Product B should be 40 per cent of the month's sales. This value is not, however, the optimum solution because by considering the amount already despatched on days previous to the decision day the 40 per cent can, at times, be reduced. Clearly, if in the extreme case 40 per cent of this month's sales had been

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despatched in the previous 4 days, the maximum that can be despatched in the next 4 days is 16 per cent because the maximum despatch in the 8 consecutive days is 56 per cent.

TABLE 5. MAXIMUM DESPATCH OF PRODUCT A ON k CONSECUTIVE DAY

Total monthly despatch (units)	k						
	1	2	3	4	5	6	7
2500	420	500	620	800	800	830	1040
3000	550	650	810	1040	1040	1060	1270
3500	670	800	1010	1270	1270	1300	1500
4000	790	950	1200	1500	1500	1530	1720
4500	920	1100	1400	1740	1740	1760	1960

Proposed Minimum Stock Rule

The rule proposed for use in the factory was to calculate Q for the month's despatch and then, if stocks fell below this value, to apply the following more stringent rule. *Calculate three new values of Q by considering in turn the despatches in the previous $n-4$ days ($n = 5, 6, 7$), choose the minimum, Q_0, and use this as the criterion for starting production.*

In practice, the month's despatch will not be known in advance and a forecast must be used. To allow for errors, this forecast is increased by 10 per cent, and the value of Q_0 is based on this higher figure. It is not possible to find the exact level of protection afforded by this scheme, but it was found, for example, with Product B that the maximum despatch in four days only exceeded that given by $\hat{Y}_4$ twice in four years. This would cause a run-out only if it occurred on one of the two days in a cycle when stocks are at a minimum.

OPERATING THE PLAN

The optimum production plan requires 21 runs on Product A and 11 on Product B each year, whereas the normal practice at the time was to have between 6 and 8 runs on each product. Because the change was so great it was decided to implement the solution in two stages, and as a first stage to have 12 runs on each product per year. The main reason for not going straight to the optimum was that it was felt that with 18 more start-ups and change-overs, requiring 36 more working days, there might be insufficient spare time to allow for breakdowns or for any unexpected increase in sales. In addition, the 36 working days would be lost for the minor maintenance work normally carried out by the plant operatives when the plant was silent. Some of this work would then have to be carried out by the factory maintenance crew, thus invalidating the assumption that labour costs are constant. It is of interest to note the cost of this decision in terms of its effect on T; it can be seen from

Table 6 that 85 per cent of the possible savings are achieved by working with this compromise solution.

TABLE 6. COST OF ALTERNATIVE SCHEMES

Scheme	Number of runs (p.a.)		Annual cost, T (£)
	Product A	Product B	
Optimum (2 runs of A per cycle)	21	11	4806
Optimum (1 run of A per cycle)	17	17	5002
Compromise	12	12	5245
Previous	6	6	7752

Construction of a Production Plan

In this section a brief description is given of how the plan would be operated. The plan to be put into operation assumes that run lengths are based on a fixed cycle length of 30 days.

The first quantity to be found is the average start-up level Q_0. The second quantity to be calculated is the expected minimum stock. During the period between deciding to start production and the material being ready for despatch, stocks will fall by an amount which on the average will be 3·5 days' sales, giving an "expected minimum stock" which is lower than the start-up level Q_0 by an amount equal to 12 per cent of the monthly sales.

TABLE 7. SCHEDULING RULES

PRODUCT A Production rate: 300 units/day			PRODUCT B Production rate: 200 units/day		
Forecast monthly sale (units)	Minimum stock (units)	Run length (days)	Forecast monthly sale (units)	Minimum stock (units)	Run length (days)
2400	450	8·0	600	160	3·0
2600	520	8·7	700	190	3·5
2800	590	9·3	800	220	4·0
3000	660	10·0	900	240	4·5
3200	730	10·7	1000	270	5·0
3400	800	11·3	1100	300	5·5
3600	870	12·0	1200	330	6·0
3800	930	12·7	1300	350	6·5
4000	1000	13·3	1400	380	7·0
4200	1070	14·0	1500	410	7·5

The run length is calculated as the number of days required to manufacture the forecast sales during the cycle. Thus, with a 30-day cycle (which can correspond to a calendar month), if the sales forecast for Product A were 4000 units, the run length (at a production rate of 300 units per day) would be 13·3 days. During this time sales would be, on the average, $4000 \times 13{\cdot}3/30 = 1780$ units, and stocks would build up to $1000 + 3990 - 1780 = 3210$ units.

This information has been summarized in Table 7, assuming a 30-day cycle. If a fixed cycle of 30 days is not used, the two halves of the table corresponding to each product are not independent.

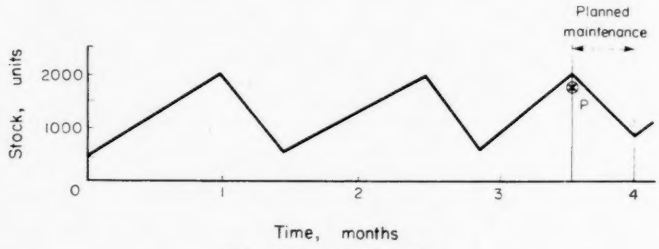

FIG. 6. *Production plan.*

When sales estimates for the following months have been obtained, Table 7 enables an optimum schedule to be drawn up, and a chart can be drawn to show the expected stock levels during the following months. Such a plan would be of the form shown in Figure 6. (In this example stocks must be greater than 1500 units at *P* in order to cover a planned maintenance period; see below.)

When operating the plan, actual stock levels will, of course, deviate from the planned stock levels, owing to variations in the sales demand and in the production rate. The way in which these deviations should be dealt with when building up stocks for a planned shut-down (e.g. for maintenance or for the August holiday) is discussed below.

At other times this variation presents no real difficulties in practice. In general, production planning can be carried out by applying the start-up rule and the run length independently. Only when changing from one product to another may the two rules conflict. In this case first priority must go to making sure that there is no stock shortage. Thus when the stock level falls to the basic danger level, preparations must be made to change over to the other product.

Stock Levels before a Planned Shut-down

When a planned shut-down is near, stocks have to be built up to a level that will cover the maximum likely sales during that period. It is not difficult to draw up a plan that has this property, by using the forecast sales, even

though the optimum conditions cannot be adhered to strictly. The difficulty arises in ensuring that stocks will be sufficient at the shut-down, no matter how the sales fluctuate beforehand.

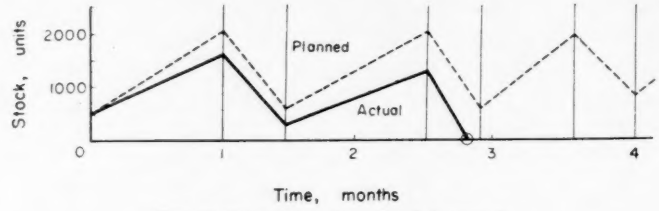

FIG. 7. *Operating plan: fixed dates rule.*

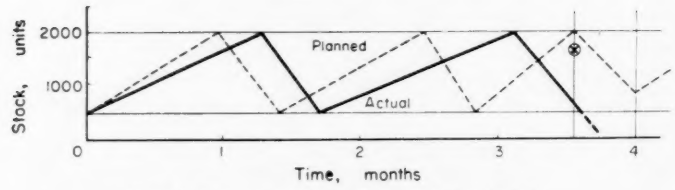

FIG. 8. *Operating plan: fixed stock levels rule.*

The first step is to calculate the required stock level P for each product at the beginning of the shut-down. This quantity must not be less than the maximum expected demand over the period of the shut-down (including the starting-up period, until the product is available for despatch). For a shut-down of up to seven days, the data of Tables 4 and 5 can be used, but allowance must be made for errors in the sales forecasts. For longer periods a further analysis of despatch data is necessary. It has been found, for example, that the despatches during the first 14 days of the month do not exceed 65 per cent of the total despatch for the month. (This figure does not apply where there is a Bank Holiday in the second half of the month.) It is then not difficult to determine the number of runs, and hence draw up a stock level plan as in Figure 6.

Variations in sales demand will cause the actual stock levels to deviate from the forecast line. Several possible courses of action are open. First, production can be started and stopped on the days originally planned. Such a course of action is unjustified at this factory, where there is rarely any need for a tight time schedule, and in addition it can easily lead to shortages as demonstrated in Figure 7. A second course of action, which is in fact the one often used, is to work between the planned stock levels as shown in Figure 8. Although this is probably adequate for most purposes, it can be seen to be of little use when attempting to meet a stock requirement on a fixed date.

The system proposed here is basically to change over when the actual stock level line meets the planned stock level line, as in Figure 9. (The various rules

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given above can be applied to avoid running out of stock.) This has a number of advantages:

(i) Errors are not cumulative, so that the probability of not reaching *P* on the appropriate date is little more than is inevitable once the last production run has started.

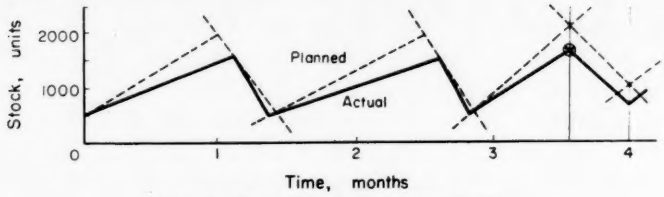

FIG. 9. *Operating plan: proposed rule.*

(ii) Run lengths are changed in the direction of the optimum, e.g. an increase in demand increases the run length.

(iii) The system is not too sensitive to short-term variations in demand.

TRIAL OF PROPOSED PLAN

A "dry-run" of the plan was made by applying the rules for starting-up with a 30-day cycle to the actual despatches from 1 April to 31 August, 1957. In this trial average stocks of the two products were found to be lower than actual average stocks by £15,000, and it confirmed that the estimate of annual savings of over £2000 is realistic. The trial also demonstrated that the plan is workable, both with regard to the application of the rules and in enabling stocks to be built up to cover the annual shut-down.

Establishing Controls over the Solution

A solution to a problem remains a solution only as long as the uncontrolled variables retain their values and the form of relationship between the variables remains unchanged. For this reason it is important to note that the results presented in this paper apply when the production rates, sales rates and costs have the values they had at the time of the investigation. The effect of changes in these values must be considered carefully. The dependence of the optimum run and cycle lengths on these parameters has been demonstrated in Tables 1–3. If a cycle length of 30 days is used, it will only need revision if the optimum length exceeds 30 days.

The start-up rule is applicable for sales rates between 2500 and 4250 units per month for Product A and between 600 and 1200 units per month for Product B.

A more important point, however, is that the appropriate inventory interest rate may change from time to time. Such a change may be brought about through changes in the general economic situation, making it possible, for

example, to borrow money in the form of a bank overdraft at a 5 per cent interest rate.

CONCLUSIONS

In this paper solutions have been presented to three problems in scheduling production on a chemical plant (i.e. run length, minimum stock levels, and planning stock before a shut-down).

The work on run lengths has shown that a substantial reduction can be made in stocks at the expense of an increase in operating cost. The run lengths necessary to give the optimum balance between inventory costs and operating costs are very much shorter than those in current practice, and it was recommended that initially, as a compromise, there should be one run of each product per month. When the success of this scheme has been established, or if further costs come to light under these new operating conditions, the question of working at the proposed optimum can be reconsidered.

The start-up rule is not expected to give any substantial reduction of stocks, but it does give a systematic and rational procedure for deciding when there is a real danger of running out of stock. The rules demonstrated in this paper are of a relatively simple nature.

ACKNOWLEDGEMENT

The authors wish to thank the Directors of The Distillers Company Limited for permission to publish this paper.

APPENDIX

Derivation of Formulae

The normal cycle of operations is as follows: Start up on, say, Product A; run on A for p_1 days; change over to Product B; run on B for p_2 days; shut down for $s-4$ days (this makes the total "silent" period s days, since the start-up and change-over take 2 days each). Assuming constant sales rates S_1 and S_2 units/day, the total stock levels will vary during one cycle as shown in Figure 1. (All symbols are listed in a glossary at the end of this Appendix.)

From the geometry of Figure 1 it can be seen that the average stocks are $\frac{1}{2}p_1(P_1-S_1)$ units of A and $\frac{1}{2}p_2(P_2-S_2)$ units of B. The annual start-up cost is $365C/r$, where C is the total of the start-up, change-over and shut-down costs for the cycle of length r days, and the total annual variable cost T is given, therefore, by the equation

$$T = 365C/r+\tfrac{1}{2}I_1 p_1(P_1-S_1)+\tfrac{1}{2}I_2 p_2(P_2-S_2),$$

where I_1 and I_2 are the costs of storing one unit of Products A and B respectively for one year.

Now $p_1 P_1 = rS_1$ and $p_2 P_2 = rS_2$, because the amounts produced during the periods p_1 and p_2 must satisfy the sales demand for the whole cycle r. If $S_1/P_1 = x_1$ and $S_2/P_2 = x_2$, then $p_1 = x_1 r$ and $p_2 = x_2 r$. Substituting for p_1 and p_2 the equation for T becomes

$$T = 365C/r+\tfrac{1}{2}r\{I_1(P_1-S_1)x_1+I_2(P_2-S_2)x_2\}.$$

Differentiating with respect to r and equating the derivative to zero gives the following expression for the optimum cycle length (r_0):

$$r_0 = \sqrt{\frac{730C}{I_1 P_1(1-x_1)\,x_1 + I_2 P_2(1-x_2)\,x_2}}.$$

The optimum run lengths (p_{10} and p_{20}) are given by

$$p_{10} = r_0 x_1 \quad \text{and} \quad p_{20} = r_0 x_2.$$

Split Runs

More complicated cycles of operation than that considered in the previous section are possible. An arrangement involving two runs of Product A to every one of Product B is illustrated in Figure 2. This arrangement was suggested by the fact that the numerical calculations using the above formulae lead to stocks of Product A that are about twice the value of those of Product B. The length (s_1-2) of the first "silent" period must not be less than two days, the time taken to change over from B to A.

The average stocks per cycle are now

$$\frac{1}{r}\{\tfrac{1}{2}p_{11}(P_1-S_1)\,(p_{11}+p_2+s_1)+\tfrac{1}{2}p_{12}(P_1-S_1)\,(p_{12}+s_2)\}$$

units of A and $\frac{1}{2}p_2(P_2-S_2)$ units of B, where $r = p_{11}+p_2+p_{12}+s_1+s_2$ is the cycle length. If C^1 is the total start-up, change-over and shut-down cost for one cycle, then the total annual variable cost is

$$T = \frac{365C^1}{r} + \frac{I_1}{2r}\{p_{11}(P_1-S_1)\,(p_{11}+p_2+s_1)+p_{12}(P_1-S_1)(p_{12}+s_2)\} + \frac{I_2}{2}p_2(P_2-S_2).$$

Now $p_{11}(P_1-S_1) = (p_2+s_1)\,S_1$, $p_{12}(P_1-S_1) = s_2 S_1$ and $p_2 P_2 = rS_2$. If $S_1/P_1 = x_1$ and $S_2/P_2 = x_2$, then $p_{11}+p_{12} = x_1 r$ and $p_2 = x_2 r$. Let $p_{11} = x_1\alpha_1$ and $p_{12} = x_1\alpha_2$; then $r = \alpha_1+\alpha_2$. Hence

$$T = \frac{365C^1}{\alpha_1+\alpha_2} + \frac{I_1}{2(\alpha_1+\alpha_2)}(P_1-S_1)\,x_1(\alpha_1^2+\alpha_2^2) + \frac{I_2}{2}(P_2-S_2)\,x_2(\alpha_1+\alpha_2).$$

Differentiating with respect to α_1 and α_2 and equating the derivatives to zero gives

$$-\frac{365C^1}{r^2} + \tfrac{1}{2}I_1(P_1-S_1)\,x_1\left(1-\frac{2\alpha_2^2}{r^2}\right) + \tfrac{1}{2}I_2(P_2-S_2)\,x_2 = 0,$$

$$-\frac{365C^1}{r^2} + \tfrac{1}{2}I_1(P_1-S_1)\,x_1\left(1-\frac{2\alpha_1^2}{r^2}\right) + \tfrac{1}{2}I_2(P_2-S_2)\,x_2 = 0.$$

Hence $\alpha_1 = \alpha_2 = r/2$, so that $p_{11} = p_{12} = \frac{1}{2}x_1 r$, i.e. the two runs on Product A will be of equal length. Therefore

$$-\frac{365C^1}{r^2} + \tfrac{1}{2}\{\tfrac{1}{2}I_1(P_1-S_1)\,x_1 + I_2(P_2-S_2)\,x_2\} = 0$$

or

$$r_0 = \sqrt{\frac{730C^1}{\frac{1}{2}x_1 I_1(P_1-S_1)+x_2 I_2(P_2-S_2)}};$$

thus $s_1 = r_0\{\frac{1}{2}(1-x_1)-x_2\}$, $s_2 = \frac{1}{2}r_0(1-x_1)$ and $p_2 = x_2 r_0$.

The above equations give the optimum if $s_1 \geq 4$. If not, the optimum can be obtained by first imposing the constraint $s_1 = 4$, i.e. an immediate change-over from B to A. The total annual variable cost is then a function of r only.

A variant of the above would be to take $s_1 - 2 = 2$; then stocks of A will not have fallen to zero. This procedure clearly leads to increased inventory costs (see dotted line) without reducing operating costs, and therefore this case will not be considered.

A more general arrangement is to have n runs of A and one run of B per cycle. If the start-up cost for A is C_1 and that for B is C_2, total start-up cost per cycle is $nC_1 + C_2$. Assuming that the stocks of A fall to zero before either beginning another run on A or changing over to B, the average stocks per cycle are

$$\frac{1}{r}\left\{\tfrac{1}{2}p_{11}(P_1 - S_1)(p_{11} + p_2 + s_1) + \tfrac{1}{2}(P_1 - S_1)\sum_{i=2}^{n} p_{1i}(p_{1i} + s_i)\right\} \text{ units of A}$$

and $\frac{1}{2}p_2(P_2 - S_2)$ units of B, where $r = p_{11} + p_2 + s_1 + p_{12} + s_2 + \ldots + p_{1n} + s_n$ is the cycle length. Then the total annual variable operating cost is

$$T = \frac{365(nC_1 + C_2)}{r} + \frac{I_1}{2r}\left\{p_{11}(P_1 - S_1)(p_{11} + p_2 + s_1) + (P_1 - S_1)\sum_{i=2}^{n} p_{1i}(p_{1i} + s_i)\right\}$$

$$+ \frac{I_2}{2}p_2(P_2 - S_2)$$

where $p_{11}(P_1 - S_1) = (p_2 + s_1)S_1$, $p_{1i}(P_1 - S_1) = s_i S_1$, $i = 2, \ldots, n$, and $p_2 P_2 = rS_2$. Now $p_{11} + p_{12} + \ldots + p_{1n} = x_1 r$ and $p_2 = x_2 r$. Let $p_{11} = x_1\alpha_1$, $p_{12} = x_1\alpha_2$, $p_{1n} = x_1\alpha_n$: then $r = \alpha_1 + \ldots + \alpha_n$. Hence

$$T = \frac{365(nC_1 + C_2)}{\alpha_1 + \ldots + \alpha_n} + \frac{I_1}{2(\alpha_1 + \ldots + \alpha_n)}\left\{(P_1 - S_1)x_1\sum_{i=1}^{n}\alpha_i^2\right\} + \frac{I_2}{2}(P_2 - S_2)x_2\sum_{i=1}^{n}\alpha_i.$$

Keeping n fixed, differentiating with respect to $\alpha_1, \ldots, \alpha_n$ and equating the derivatives to zero shows that $\alpha_1 = \alpha_2 \ldots = \alpha_n = r/n$. Thus

$$T = \frac{365(nC_1 + C_2)}{r} + \frac{I_1}{2}\left\{(P_1 - S_1)x_1\frac{r}{n}\right\} + \frac{I_2}{2}(P_2 - S_2)x_2 r.$$

Treating n as a continuous variable, differentiating with respect to r and n and equating the derivatives to zero, leads to the following equation:

$$n_0 = \sqrt{\frac{I_1 P_1 x_1(1 - x_1)C_2}{I_2 P_2 x_2(1 - x_2)C_1}}.$$

To find the actual optimum value, n_0 may be approximated by a simple fraction n_1/n_2. Then a cycle can be defined as having n_1 runs of Product A and n_2 of Product B. It then remains to test whether the start-up and change-over periods make the system feasible.

GLOSSARY

C	$C_1 + C_2$ (£)
C^1	$2C_1 + C_2$ (£)
C_1	Cost of starting up and shutting down on A (£)
C_2	Cost of starting up and shutting down on B (£)
D	Total monthly despatch (units)
I_1	Cost of storing one unit of A for one year (£/unit-year)
I_2	Cost of storing one unit of B for one year (£/unit-year)
n	Number of runs on Product A per cycle
n_0	Optimum number of runs on Product A per cycle
P_1	Production rate for Product A (units/day)
P_2	Production rate for Product B (units/day)

p_1	Length of run on Product A (days) when only one run per cycle
p_2	Length of run on Product B (days) when only one run per cycle
p_{10}	Optimum run length for Product A (days)
p_{20}	Optimum run length for Product B (days)
p_{1i}	Length of run on Product A when n runs per cycle (days)
Q	Basic danger level (units)
Q_0	Smallest value of Q given by minimum stock rules (units)
r	Cycle length (days)
r_0	Optimum cycle length (days)
S_1	Sales rate for A (units/day)
S_2	Sales rate for B (units/day)
s	Length of silent period (days) when only one run per cycle
s_i^{-2}	Length of ith silent period (days)
T	Total annual variable cost (£/year)
x_i	S_i/P_i
Y_k	Maximum despatch in k consecutive days (units)
α_i	p_{1i}/x_1

Automatic Data Collection and Analysis for Works Investigations: Early Experience with an Automatic Performance Recorder*

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Richard Thomas & Baldwins Limited

For Operational Research studies concerned with process operations, the collection of adequate data is necessary for the formulation of a model; subsequently, further data are required for the testing of the solution and finally the routine control of the solution will necessitate the setting-up of a permanent system of data collection and analysis. The necessity to improve on existing methods of data handling for all these purposes led to the development of an Automatic Performance Recorder. The basic logic of this equipment is described together with some practical examples of past and potential applications.

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INTRODUCTION

THERE are many phases to an operational research study but very rarely are papers written on more than two, namely, the derivation of a model and its manipulation. Adequate and accurate data are an obvious pre-requisite for any operational research study, and our experience with steelworks problems has shown that there is scope for improvement in methods of data collection and analysis, both for investigational work and for permanent routine purposes.

It is, then, the intention of this paper to show how, in our company, the need for some form of automatic data-recording equipment became apparent, and to describe past and potential applications of the equipment which we acquired.

THE CASE FOR AUTOMATIC RECORDING EQUIPMENT

In 1957 the Operational Research Department was asked to carry out a trouble-shooting study in a department in which there were five similar plate-coating machines. The reason for the request was that production per machine in the shop was far below that expected. To obtain a detailed picture of events in the shop, observers were put on one machine for a week and then on a second machine for another week. In both cases output rose by about 30 per cent. An attempt was then made to ascertain from production records the reason why the machines did not normally operate at this level of production.

The rate of output while a machine is running is dependent on machine speed, the dimensions of the plates being processed, and on the gaps between successive plates. Assuming an average machine speed (the one observed), the percentage of the total time accounted for by actual processing time was

* Based on a Paper presented to a meeting of the Operational Research Society on 21 October, 1959, by S. L. COOK.

calculated for several past weeks and for different machines. The percentage accounted for by recorded delays and reprocessed material was then added on and the totals obtained compared with the expected 100 per cent. The results of this analysis are shown in Figure 1.

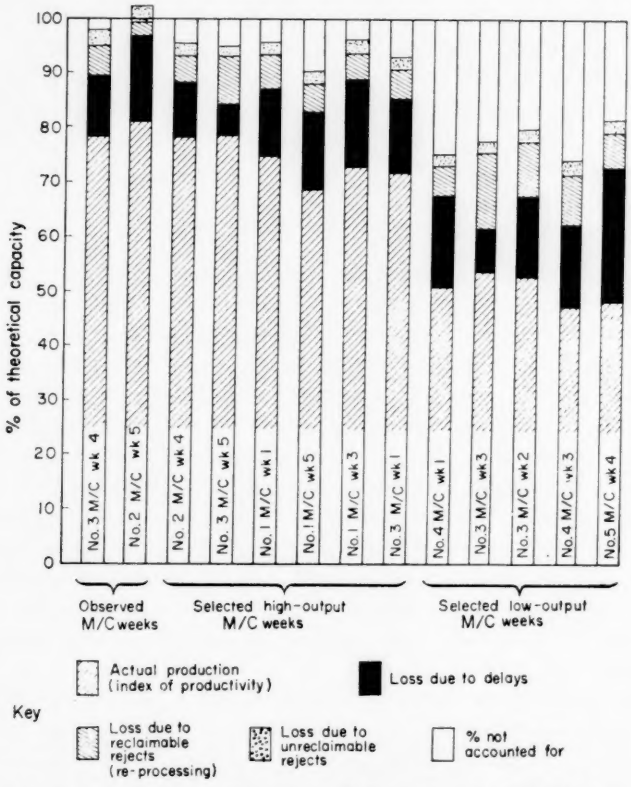

FIG. 1. *Coating-machines. Comparison of observed, and selected high and low output, machine weeks.*

It can be seen that for the observed machine-weeks the agreement between actual and expected values was quite close. For unobserved machine-weeks of high output there was still fairly good agreement, though the time unaccounted for was between 3 per cent and 10 per cent of the total. For machine-weeks of low output the time unaccounted for ranged from 20 per cent to 25 per cent. It was interesting to note that for these weeks the recorded delays and re-processing times were very little greater than for the good output weeks.

From this study it was concluded that:

(1) The production records being kept did not indicate the real causes of lost production.

(2) A special instrumented study of factors affecting machine speeds and performance should be made.
(3) Machine performances and particularly machine speeds should be recorded as a routine.
(4) The possibility of installing equipment which would record machine performance automatically, preparing shift and weekly production records, should be investigated.

On completion of this study, further consideration confirmed the potential advantages of automatic data recording: the problem then remained of determining what available equipment was best suited, or could be developed, for this and other possible studies.

SELECTION OF EQUIPMENT

Automatic data-recording equipment was needed to meet two main requirements. Firstly, it had to be sufficiently flexible for use in a wide variety of works investigations; and secondly, it had to be capable of demonstrating to managers, accountants and others the possibilities of automatic recording in the production of those records necessary for their respective forms of control. For production records to be of maximum use, they should be available as quickly as possible after completion of the period to which they refer, and this makes accumulation of information within the equipment desirable.

With these factors in mind several types of commercially available equipment were considered but none was considered to be entirely suitable. Some produced data requiring manual analysis; others, being continuously running, produced large quantities of tape and did no intermediate analysis, making it inconvenient for production or accounting reports to be obtained on a routine basis.

The machine which came nearest to meeting the requirements was a specially designed recording system which had been installed at Messrs. Guest Keen and Nettlefolds Limited. This equipment had been developed jointly by the British Iron and Steel Research Association and Digital Engineering Limited, and was intended to serve as a performance recording system for 18 machines in one shop. The data obtained were summarized on punched tape for routine analysis. Although this installation was not intended for use in research work, the equipment used appeared to meet the basic requirements. Consultation with the manufacturers confirmed that the development of a suitable automatic performance recorder was feasible, and a specification was agreed upon.

THE PERFORMANCE RECORDER

The design of the Recorder is sufficiently flexible to allow its application to a variety of processes and its important features are the automatic timing and counting functions, and that the information is punched out on paper tape.

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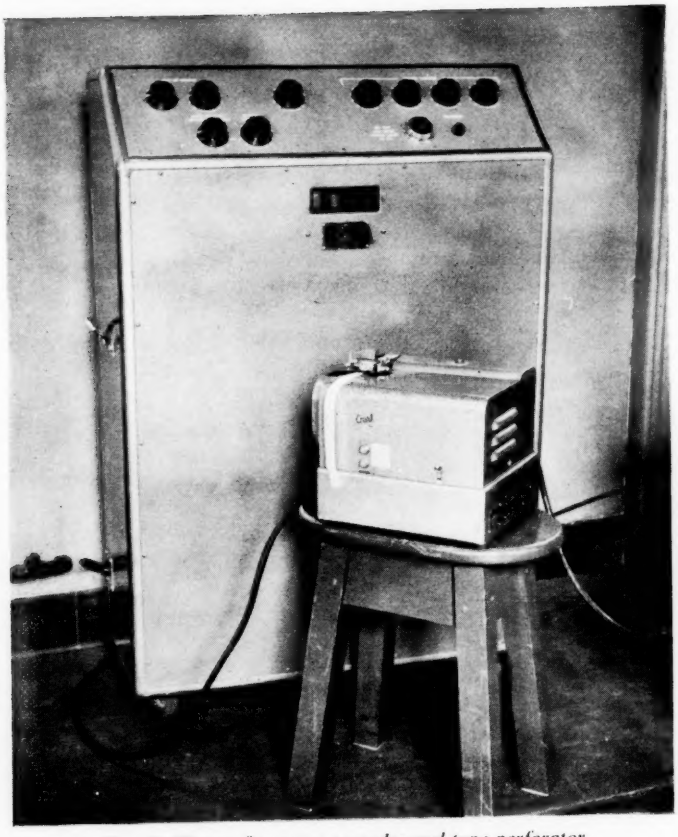

FIG. 2. *The performance recorder and tape perforator.*

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For investigational purposes the data can be rapidly analysed directly by an electronic computer or via punched cards. For demonstrating the feasibility of a permanent installation the data can be printed out immediately by use of an electric typewriter or further summarized by computer or punched cards to produce the necessary reports. An illustration of the Recorder is given in Figure 2, and the control panel is shown in Figure 3.

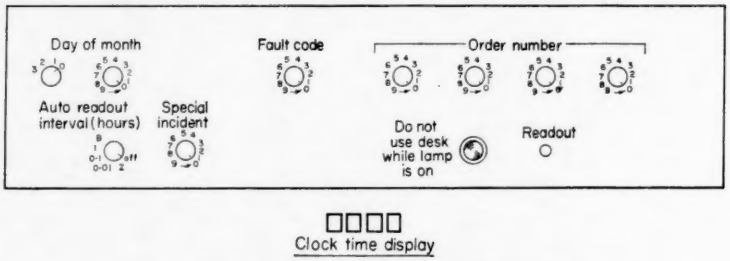

FIG. 3. *Control panel on performance recorder.*

The Recorder accumulates information on 15 channels by means of its pulse-counting and register functions, the basic logic of these being shown in Figure 4.

Seven counters with a total allocation of 24 digits accumulate external electrical impulses which can be generated manually or automatically. Manual pulse-generation by means of pushbuttons may be expedient for comparatively few and infrequent pulses, but most applications require automatic pulse-generation. For this transducers of various types—mechanical contact, photo-electric, magnetic, etc.—are required for transforming physical quantities into electrical impulses. It is sometimes necessary to switch pulses from a common source into a number of counters; for example, pulses generated by plates leaving a coating machine will require to be switched to the channel corresponding to the category into which the plates are classified, such as primes, seconds, etc. Switching can be manual by means of an external rotary switch, or automatic by using electrical relays.

For recording running and idle time of a machine, two 5-digit counters are used to accumulate pulses from an internal clock. These pulses occur at 0·001 hour intervals, and are automatically switched to the appropriate counter by means of a relay.

The six channels of the register function have a total allocation of 14 digits. Three of these channels, namely clock-time, shift number, and day of month, operate automatically following the initial setting-up at the start of a trial. The 4-digit Order Number register is set manually by means of four rotary switches to identify the material being processed, but it is sometimes more convenient for subsequent data analysis to use these switches for recording physical characteristics such as a dimension or steel grade of the product.

The 1-digit Fault Code is used to record the reason for a delay, and use of the 1-digit Special Incident register is referred to later.

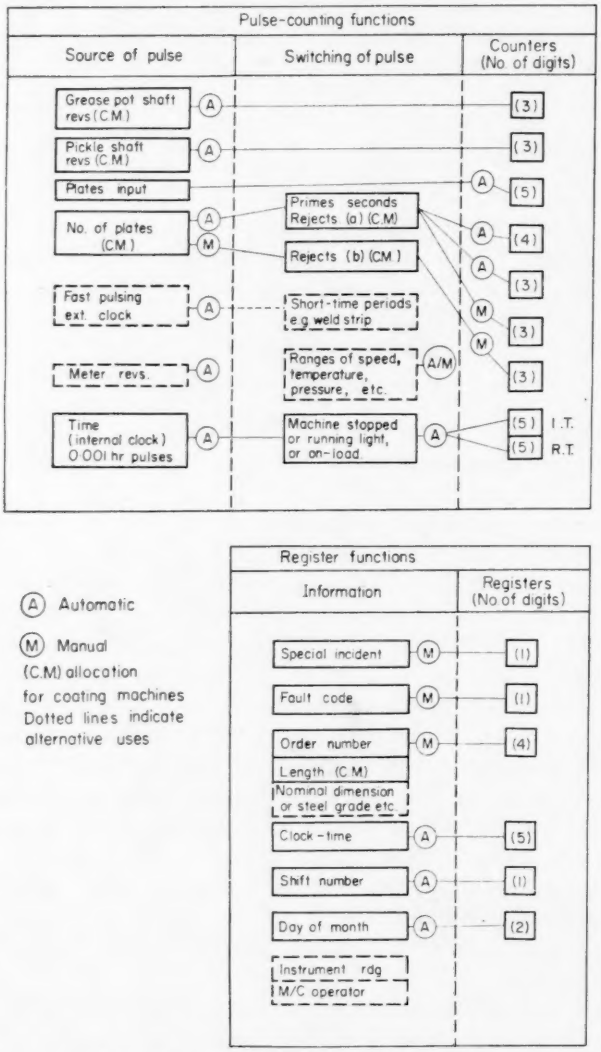

FIG. 4. *Functional logic of performance recorder.*

The 15 channels of the Recorder comprise a total of 48 digits, and when a readout is initiated these are scanned in turn and the accumulated information is punched out on 5-channel paper tape.

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Readouts can be initiated manually by pressing the readout button, or automatically by use of the auto-readout switch. When this switch is on its "Z" position the Recorder will read out whenever the machine being studied stops and/or starts. Other positions of the switch give readouts at intervals of eight hours, one hour, one-tenth of an hour and one-hundredth of an hour. Also, in order that no external-pulse counter shall overflow and information be lost, a readout is initiated automatically whenever any counter reaches its capacity. All these readouts (manual and automatic) may be called "normal" in that following a readout the running and idle time and external-pulse counters reset to zero.

A rather different kind of readout is obtained by using the Special Incident Switch, and this is used to indicate that a certain event occurred at a certain time, the counter readings at that time being included on this readout. The switch is inactive on its 0 position, but initiates a readout when the code number (1–9) for a certain event is set. Counters are not reset to zero following a Special Incident readout, and therefore these readouts can be processed separately. Alternatively, the Special Incident switch can be arranged for use as a second Fault Code when the number of reasons for a delay exceeds nine.

During actual operation it is necessary for an observer or the machine operator to attend the Recorder. The order number switches are set up manually and must be kept up to date with the order being worked. When there is a delay the fault code switch is set to the number (1–9) appropriate to the reason for the delay.

Three methods of automatically processing the data tapes are possible. These are:

(1) Periodic printing-out of raw data on a tape-reading electric typewriter.

(2) Automatic conversion to punched cards. This is useful when the sorting and tabulating of moderately large amounts of data is necessary, but will depend upon the accessibility of punched-card equipment.

(3) Electronic computer. This method is essential when large quantities of data require to be rapidly summarized or statistically analysed. The Ferranti numerical code chosen for the data tapes enables the raw data to be fed into a Ferranti computer without intermediate processing, but if necessary, the data can be transcribed to suit any computer.

To date, the Performance Recorder has been used on a Hot Strip Mill and a description of this application and method of analysis follows:

Hot Strip Mill Application

The Hot Mill is a composite unit of three main sections: cold 5-ton steel slabs are firstly pushed through one of three *Reheating Furnaces*; the hot slabs then pass through the four separate stands of the *Roughing Mill*, and finally through the 6-stand tandem *Finishing Mill*. Slabs are now called "pieces", and

on leaving the mill these are either coiled or hot-sheared into multiples of final length and piled.

Data on hot mill operation were required for profitability studies, and in view of the repetitive nature of the operations and the quantities of data which would require subsequent processing this was thought to provide a suitable application for the Performance Recorder.

This being the first actual application of the Performance Recorder, a moderately ambitious scheme was devised in which the required data were obtained from the sixth Finishing Stand on two types of readout.

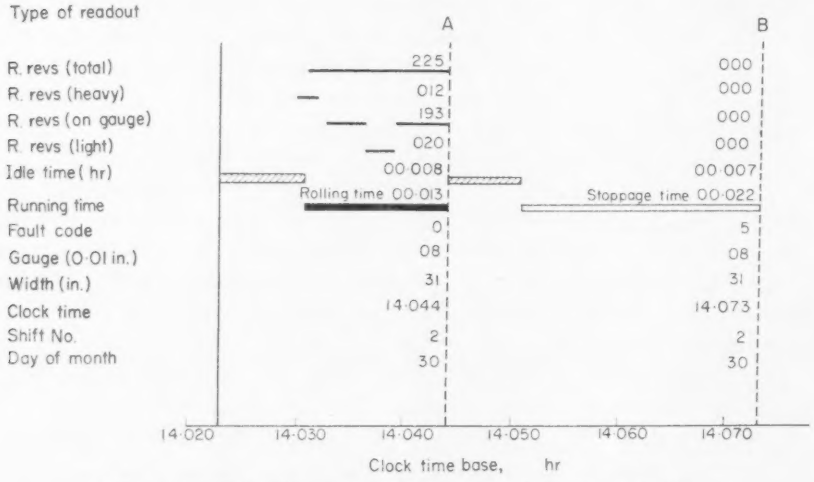

FIG. 5. *Examples of types A and B readouts. Hot mill application.*

Type A readouts were initiated automatically as each piece left the mill, and recorded the following information for each piece rolled:

(i) Idle time (time from departure of previous piece to arrival of current piece).
(ii) Running time (time that piece was actually in the mill).
(iii) Total roll revolutions during time that piece was in the mill.
(iv) Separate number of roll revolutions when output thickness of strip was within gauge tolerance, off-gauge heavy and off-gauge light.
(v) Nominal gauge and width.
(vi) Clock time, shift number, and day of month.

This information was automatically recorded, except for the initial setting of clock-time and day of month, setting of nominal gauge and width on the four Order Number switches at the start of each order, and the manual switching of roll-shaft pulses into one of three channels according to the indicated gauge. Automatic switching for gauge will be possible for a future application.

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Type B readouts were initiated automatically when the mill restarted after a stoppage. During the stoppage period, the observer set the code number

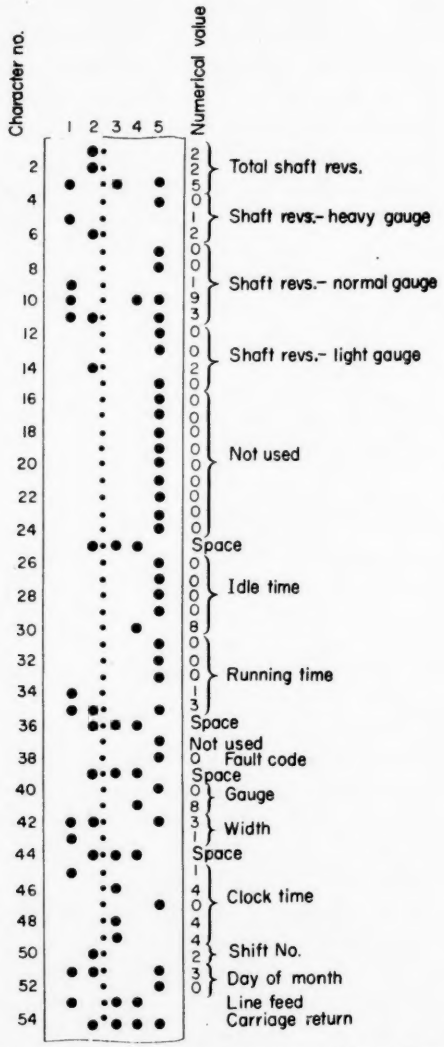

FIG. 6. *Specimen of tape for type A readout. Hot mill application.*

corresponding to the reason for the stoppage on the Fault Code Switch, and this number was then included in the readout. The duration of the stoppage period was recorded on the Running Time accumulator, but these readouts

were identifiable (to avoid confusion with type A readouts) by means of the Fault Code Number, and by their zero roll revolutions.

Figure 5 illustrates one Type A and one Type B readout. Figure 6 shows a sample Type A readout on tape. It will be noticed that idle time (mill still running) and stoppage time were recorded separately on Type B readouts. The external circuitry included a pulsing contact on the roll shaft, a load relay energized when a piece was in the mill, and a voltage relay which closed a contact when the mill stopped turning. The load relay switched the internal clock pulses to the Running-time accumulator, and also allowed the roll-shaft pulses to enter the Recorder. The voltage relay, being in parallel with the load relay, switched the clock pulses to the Running-time accumulator when the mill stopped.

The data tapes from this application were analysed on the B.I.S.R.A. Pegasus Computer. The analysis consisted mainly of sorting the Type A readouts into gauge/width groups, and computing for each group the average rolling and between piece time, average length and speed, and percentage of on-gauge and heavy or light off-gauge. Type B readouts were sorted to give the total time for each kind of delay.

The results of the analysis were used to determine the mill production rate of each combination of gauge and width, and to examine such relationships as the effect of gauge or width on production rate or speed, nominal gauge on percentage off-gauge, etc. These and many similar relationships have eventually to be established for different grades of steel and for different operating crews and practices generally, in order on the one hand to calculate the capacity and profitability of the mill for different kinds of orders, and on the other hand to determine the best operating methods with a view to improving production.

Coating-machine Application

As mentioned earlier, it was the result of studies on coating-machines that initiated the development of the Performance Recorder, but the application has not yet been put into practice. The proposed method of using the Recorder is intended to yield data for investigational work and to allow the feasibility of a permanent data-recording system to be studied.

For practical purposes, the main components of the machine are the pickler and the integrated coating and greasepot unit.

Output of the line is measured in terms of area of coated plate produced, but for any particular length and width of plate processed the output is determined by the greasepot speed, which largely depends on quality requirements, and by the amount of gap between consecutive plates in the greasepot.

A certain minimum gap is essential, and this can be controlled by adjusting the pickle shaft speed relative to the greasepot speed.

The functions of the Performance Recorder in this application will be to count the number of input plates, the number of coated plates coming off the

line in each of four quality categories, the number of revolutions made by the greasepot and pickler drive shafts, to record the running-time periods over which these counts were made, and to record the duration of and reasons for delays. The four Order Number switches would be used to record the unit length of the plate being processed.

Item No.	Item	Decimal digits	Maximum recorded number	Method of actuation
1	Greasepot shaft revolutions	3	999	Automatic
2	Pickle-shaft revolutions	3	999	Automatic
3	Number of input. (Plates)	5	99,999	Automatic
4	Number of output. (Primes)	4	9,999	Automatic
5	Number of output. (Seconds)	3	999	Automatic
6	Number of rejects (a)	3	999	Automatic
7	Number of rejects (b)	3	999	Push-button
8	Idle time	5	23·999 (hrs)	Automatic
9	Running time	5	23·999 (hrs)	Automatic
10	Special incident	1	9	Hand-set. See Code
11	Reason for delay	1	9	Hand-set. See "Fault Code"
12	Length of plate in direction of travel	4	9,999	Hand set on "Order No." switches
13	Clock time	5	23·999 (hrs)	Initially hand-set, subsequent setting is automatic
14	Shift No.	1	3	Automatic
15	Day of month	2	31	Initially by hand-switch, subsequent changes automatic

FIG. 7. *Allocation of channels for coating-machine application.*

"Normal" readouts (with counters resetting to zero) would be initiated manually at the end of each order and automatically at each change-over from running to idle time. Special incident readouts would be initiated by dialling the appropriate number on the Special Incident switch when certain events occurred, e.g. shift change, new input-pile, output-pile change (primes and seconds), start of new order.

An example of a readout, showing the allocation of channels, is shown in Figure 7. Mechanical contact devices would probably be used for counting the

revolutions of the two shafts, and photo-electric transducers for counting the input plates, and output primes, seconds, and rejects (a); rejects (b) are infrequent and would be counted by means of a manually operated pushbutton.

The data would be analysed, either via punched-cards or computer, to give a detailed breakdown of the use made of the capacity of the machine by comparing theoretical and actual inputs and outputs; also to break down the loss of theoretical capacity due to delays, rejects and re-processing of menders. In addition, various relationships could be investigated, e.g. between speed and coating thickness, delays and proportion of sub-prime material, linear utilization (ratio of plate length to length plus gap) and length or speed, and many others.

Other Future Applications

Two applications of the Performance Recorder are under consideration on an Electrolytic Tinning Line. The first of these will be the measuring of the proportion of strip entering the line at various thicknesses (in terms of 0·0001 in. deviation from nominal gauge), and the other with the classification of reject material. Both are fundamentally similar to the Hot Mill and Coating Machine applications, and will not therefore be described in detail.

Further possible applications, e.g. Open Hearth Furnaces and Cold Reduction Tandem Mills, will be considered in the future, and these will undoubtedly make demands on the versatility of the Recorder. Some of the pulse-counting uniselectors will require modification to record digitized instrument readings such as inter-stand strip tension in a tandem mill, and temperature, pressure and oil, steam, or air flow on a furnace. Digitizers are available for analogue-digital conversion of instrument readings. It may be that an average reading of, say, temperature, over a period of time will be required and the best method then could be to integrate temperature with time and count in one of the pulse-counting circuits.

In addition to the 48 numerical characters which the Recorder at present reads out, there are 4 space characters. The total of 52 characters can be rearranged by fairly simple alterations to the internal circuitry and this would be necessary if, for instance, provision was required for 8 characters for identification of the material being processed in a machine.

In the event of the Recorder being used to demonstrate a possible routine application where it was not intended to station a permanent observer, normal machine-operators would provide the necessary manual inputs and some form of continuous chart could show the operator (and possibly a supervisor) that the correct information was being fed in.

LIMITATIONS OF THE PERFORMANCE RECORDER

Although the Recorder is quite flexible in many respects, it has some limitations which make it unsuitable for certain types of application. These limitations are:

Permanent Routine Data Recording

Since flexibility for investigational purposes was an important factor in the design of the Recorder, it is not itself expected to provide the most efficient nor the most economic data-recording system for any particular type of plant on a permanent routine basis. Once the feasibility of automatic equipment for a particular application has been established, suitable permanent equipment would be purpose-designed to meet the requirements of the proposed system.

Multipoint Recording

The Recorder is unable to accept information from two (or more) sets of events which occur simultaneously, but which may or may not be mutually independent. This applies both to events which occur in parallel (similar machines carrying out the same operation at the same time) or in series (work-pieces passing through several consecutive processes). Parallel recording would require a Recorder for each machine, but a queuing device would allow all the Recorders to readout through a common Tape Reperforator.

The problem of series recording is illustrated by considering a single production line comprising two machines, A and B, through both of which in turn work-pieces must pass. The inter-process time-lapse means that several pieces may pass through A during the time-interval between the first piece entering A and the same piece leaving B. To record A and B process data on the same readout as each piece leaves B would require that process A data should be held in storage pending the emergence of the piece from B. This situation arises frequently in operational research studies, where it is necessary to investigate interactions between different parts of a plant. The difficulties of comprehensive data-recording in such situations are not considered to be impossible to overcome, but additional equipment would be required.

Timing of Events of Short Duration

Since the frequency of the internal clock pulses is only one per 0·001 hours (3·6 seconds), and the readout takes up most of the time between two successive clock pulses, any external pulses entering the Recorder during that time will be lost. Events requiring more accurate timing could be recorded by means of an external fast-pulsing clock if there was time between these events for a readout.

Normally, the Recorder is considered satisfactory for recording time periods from about half-a-minute upwards; below this, the rounding error may be serious. The times involved in most operational research studies are of sufficient length to reduce this error to negligible proportions, but it is nevertheless an important consideration when small time-intervals are involved.

CONCLUSION

Data collection is an essential part of any practical operational research study. Before a model can be formulated, it is necessary to analyse process data; further data are needed to test the solution and, finally, a regular and permanent system of data collection needs to be set-up in order that the solution can be monitored. The development of automatic collection and processing of works data is, then, of great importance to operational research workers.

This paper has described a Performance Recorder which is, subject to certain limitations, adaptable for data collection on a wide variety of steelworks processes, both for investigational work and for studying and demonstrating the feasibility of permanent routine data collection systems. Routine systems will require equipment specially designed for each application, and this will enable management and accountants to be provided with the information necessary for overall control of processes.

Early experience with the recorder has indicated that it provides a feasible approach to the problem of automatic data collection. Further development along these or parallel lines is essential to meet the increasingly complex requirements of current and future trends over the whole range of industrial processes.

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ACKNOWLEDGEMENTS

For many helpful discussions in the exploratory stage, we are indebted to the Road Research Laboratory, the British Iron and Steel Research Association, Messrs. Guest, Keen and Nettlefold Limited and the Steel Company of Wales Limited. Digital Engineering Limited designed and built the Performance Recorder, and Creed Limited supplied the Tape Reperforator.

We are indebted also to the Directors of Richard Thomas & Baldwins Limited for permission to publish this paper.

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Letters to the Editor

THE last issue of the *Quarterly* contained reviews of two books on Cybernetics, a subject which should be of interest to every operational research worker. Having recently, in common with many others, completed a careful reading of one of these books, *Cybernetics and Management* by Stafford Beer, I am a little concerned lest your reviewer may have put off some people from studying it. This review gives the impression that the book is rather superficial, bears the marks of enthusiasm and does not carry conviction. Now, the reviewer is entitled to this impression and entitled to say so (we do not want the reviews in the *Quarterly* only to be congratulatory and servile) but I, for one, had a rather different reaction.

The author states clearly in his introduction that this is an expository volume; its purpose to deploy the subject matter and the methodology and to encourage the reader to share something of the author's enthusiasm by being willing to read further. So far as I am concerned he has achieved this aim. I would also hope that managers and executives would react in a similar fashion.

It is part of the author's case that this new science can only be tested and can only be developed if management is willing to attempt this fresh approach to communication and control. Your reviewer is nervous lest a stimulated desire to use cybernetics be frustrated and embittered by the immaturity of the science. However, as I understand it, part of the author's intention in writing this book is to break out of this vicious circle. While not anticipating that everyone who reads it would become an embryo cybernetician overnight, I would nevertheless encourage scientists and management alike to study Mr. Beer's case and to form their own judgement.

B. H. P. RIVETT

Roman House,
Cripplegate Buildings,
London, E.C.2.

THE paper by Holt and Salveson studies the behaviour of committee members, who contribute more or less fully and freely to the discussion according to where they sit, how much confidence they have, and their status relative to the other members.

These are the factors which condition behaviour in casual conversation, but in a committee it is surely the chairman's job to regulate the size of contributions according to their value and nothing else. The curious and interesting phenomena described in the paper reflect on the effectiveness of the chairman.

In some of the discussions described, the chairman was one of the psychologists, and so was interested in studying the committee as well as in making it work. It would be interesting to know to what extent an effective and single-minded chairman could have changed the situation.

He might have proved more valuable than sessology and intrapersonal dynamics.

F. D. ROBINSON

1 Church Road,
Alsager,
Stoke-on-Trent.

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Book Reviews

Statistical Methods in Biology.

N. T. J. BAILEY.

English Universities Press, London, 1959. 25*s*.

This book has been very well thought out so that it may serve the purpose defined in the introduction, namely, "to provide workers in the biological and medical sciences with an elementary account of the chief statistical methods liable to be needed in practice". Dr. Bailey's style is easy and his explanations are lucid so that the book should be comprehensible to scientists in many fields who wish to use the more elementary statistical tools. It is possible that the more intelligent of these scientists may on occasion wish to find out how some of the results are obtained, but the mathematical background to most of the tests has been deliberately excluded. With some of the tests discussed, the scientist will find it a case of applying rules without understanding why, which may be unavoidable at the present juncture where rigorous mathematical and statistical training of engineers and scientists is frequently lacking, but one would hope that ultimately these people will obtain, as part of their undergraduate studies, a more basic knowledge of statistical tools.

In this book there is a fairly strong case for better references at the end of each chapter. A short general bibliography is given but the reader might find it more useful to have his attention drawn, for instance, to Cochrane, Yates and Quenouille on Experimental Design specifically in relation to Chapter 11, should he wish to pursue further the notions propounded by Dr. Bailey.

In the short space at his disposal, Dr. Bailey has been unable to pursue the logical background to some of the tests described but he has been very careful to stress the necessity for calling in expert statistical assistance where problems arise—one can only hope that this advice will not be ignored or irrevocable damage may be done by the patient who self-administers "first aid" before the half dead corpse reaches the hospital or doctor.

At the end there is an ingenious section called "Summary of Statistical Formulae" included as a quick reference guide to the reader with some knowledge of statistics.

The chapters on Experimental Design, Random Samples and Random Numbers are particularly good and, if practitioners will stick to the advice given, they are unlikely to introduce biases into their experiments.

This book should have a wide sale.

S. V. CUNLIFFE

Tables of Random Normal Deviates.

J. M. SENGUPTA and NIKHILESH BHATTACHARYA.

Indian Statistical Institute. Asia Publishing House, London; Statistical Publishing Society, Calcutta, 1958. xix + 26 *pp.*, 35*s.*

Tables of random numbers, which are useful in statistical theory and in operational research in applying Monte Carlo methods, are basically random selections of the digits 0 to 9, and in that form they are probably most generally useful. If it is required to make random selections from any form of population other than the rectangular form in which each digit has equal probability, combinations of the usual random numbers have to be transformed—a procedure that is easy enough but laborious. The other distribution from which selections are most commonly made—perhaps more in statistical theory than in operational research—is the normal distribution, and there are corresponding sets of tables, of which the one under review is the latest. It consists of 10,400 randomly selected individuals from a normal population with a mean of zero and a standard deviation of 1000 units, derived by transforming the original 41,600 random numbers I compiled some 35 years ago, and representing a revision of a set published in Sankhyā in 1934. Results are given of tests of randomness of the tables, and patches where the data are—not non-random, but unusual among random arrangements, are identified. The tables are well printed and well arranged so that they are easy to read.

L. H. C. TIPPETT

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Messen und Regeln in der Betriebswirtschaft. Einführung in die informationswissenschaftlichen Grundzüge der industriellen Unternehmensforschung.

A. ADAM.

Physica-Verlag, Wurzburg, 1959. *pp.* viii + 179. *Dm.* 27.

"Introduction to . . ." means here a quick sight of what Dr. Adam happens to have in his notebooks. He is presumably a busy man and has not had time to consider for whom he was writing, or to make his notes into the shape of a book. Anyone familiar with the topics might spend a profitable afternoon reading the notes like a conversation, politely ignoring what he knows already and picking out interesting points. Unfortunately Dr. Adam himself will not be there to explain obscurities. Anyone unfamiliar with the topics will find the book virtually unintelligible.

The word "industrial" appears in the title but the notes are almost wholly abstract in form and there are very few of the bogus "practical" applications that one has come to expect. The longest chapter (20 pages) consists of notes, declared to be incomplete, on the tasks and organization of an operational research department in a firm.

The names of topics include most of the names associated with information theory, the theory of measurement, and cybernetics. There are some interesting

philosophical aperçus belonging to the epistemology of this region, which might stimulate a receptive reader. The philosophical platitudes with which the notes begin, in the stock Germanic manner, may be overlooked. The principal formal instrument is the matrix. The chapter on "the cybernetics of the firm" is short and weak.

The thing to be said in favour of slapping one's notes at the public, instead of "writing a book", is that it is a perfectly honest procedure—no padding, no exaggerated claims, no rhetoric, no job-hunting. There is more pleasure and profit in inspecting what an interesting and active mind has made a note of, than in looking for a needle of sense in a haystack of logico-mathematical publicity.

P. G. LUCAS

The Theory of Linear Economic Models.

DAVID GALE.

McGraw-Hill, London, 1960. *pp.* xvii + 330. 74*s.*

The author emphasizes in his Preface "that this is a text not in economics but in applied mathematics", although it deals with topics which concern the economist. To make this possible, typical economic situations must be described by an abstract and simplified model. "We have tried to select those models which best illustrate the manner in which mathematical reasoning can be used to obtain information about idealized economic situations." The topics he has selected are Linear Programming "which is already being used quite extensively in industrial planning", and "game theory and some of the equilibrium models, which are in no sense ready for practical application in their present stage of development". The treatment is restricted to linear theory, and "the feature of this theory which plays the unifying role in most of the applications is the fundamental notion of *duality*".

As far as the student of economics is concerned who wants to delve more deeply into the mathematical background of such books as Linear Programming and Economic Analysis by Dorfman, Samuelson and Solow, or Mathematical Economics by R. G. D. Allen, this is the book for him.

Chapters 1, and 3–5, deal with L.P., while Chapter 2: Real Linear Algebra, contains some fundamental theory of linear equations and inequalities and related geometrical concepts. The treatment starts with examples and statements of theorems about duality. It continues with rigorous proofs. One section in Chapter 3 is entitled An Application: Allocation of Resources in a Competitive Economy. This deals with a topic that has not been included in any other book on the mathematics of L.P. To give an idea of the type of problems with which it deals, we summarize the results: "For a competitive economy in which each firm operates a set of linear activities it turns out that it is possible to assign prices to resources in such a way that, although each firm acts so as to maximize its profits, the demand for resources will not exceed the available supply.

Furthermore, at these prices the firms in maximizing their own profits will automatically be operating so as to maximize the value of the total output of the economy" (pp. 92–3). "A neater mathematical argument could hardly be found to support the philosophy that everything is for the best in this best of all possible, freely competitive, linear worlds!" (p. 90). The latter quotation gives an idea of the author's chatty style.

In Chapter 4 there is a description of the Simplex Method, "one of the most celebrated computational procedures in recent mathematical history" (p. 105). Chapter 5: Integral Linear Programming, deals with Flows through Networks and the Transportation Problem (after Ford and Fulkerson), with the Assignment Problem (after Kuhn), and with similar topics. An amusing (deliberate?) *gaffe* is the inclusion in the Index of the name of R. E. Gomory. The page referred to (p. 132) does not mention him, but contains the following passage: "some highly ingenious computational devices have been invented for obtaining numerical solutions. This topic, however, is beyond the range of our present study". It is a pity that the methods of Gomory, and of others, for the solution of L.P. problems with the further constraint that some or all variables have integer values, were not sufficiently known at the time of writing the book to warrant inclusion.

Chapters 6–7 deal with two-person zero-sum games and with the solution of matrix (i.e. finite) games. Examples of infinite games are also given. The Main (Minimax-) Theorem of the Theory of Games is proved by the reduction of games to symmetric ones.

The last two chapters are about Linear Models of Exchange, and Linear Models of Production. These "have more the flavor of the problems which occur in ordinary courses of economics" (p. 260). Equilibrium problems, the Leontief Model, and von Neumann's Model of an expanding economy find their place and receive a thorough treatment. Here the present book differs once more from other mathematical texts on L.P. and game theory.

At the end of each chapter there are bibliographical notes and exercises, some simple and some more difficult, with hints but, alas, without solutions. There is also a short bibliography and an Index.

It was the aim of the foregoing remarks to convince the reader that this is a book unique in its way and a necessary addition to the collection of mathematical books of an economist or of an operational research worker.

S. VAJDA

Mathematics—Education and Industry.

A Conference of Teachers, Research Scientists and Industrialists at the University of Liverpool, April, 1959.

Proceedings published by *Technology*.

Operational research groups flourish on a mixture of academic disciplines but to a great extent the unifying link is provided by Mathematics. A conference

entitled "Mathematics—Education and Industry" is thus of interest to all concerned with the future supply of operational research recruits and it was not surprising to find a number of operational research groups represented among the twenty resident industrialists.

As the proceedings show, practically all the ninety resident teachers were well informed about the place of mathematicians in industry as "model builders" and not simply as "problem solvers". There was also a general familiarity with (and healthy contempt for) the capabilities of electronic computers, not only among the university and college lecturers but also among the schoolteachers.

Against this favourable background, one would have expected a similar knowledge of the work and prospects of Operational Research. Alas no! The subject was left until practically the end of the conference before getting any official mention in the programme although sessions on linear programming, mathematical statistics and computer Monte Carlos were included earlier. Judging from many informal discussions, the fact that Operational Research is the one activity which practises all of these and certainly the main place for most of them, was only slowly being realized. Even if the teachers who attended the conference are now better informed, what of those who did not?

This report of the conference covers, in 150 pages, the lectures and discussions of a week. It does so extremely well and conveys the balance struck between purely domestic educational subjects such as "The measures needed to provide the mathematical specialists likely to be required in the next 10 years," expositional talks on the place of mathematics in about ten different industries and intermediate topics of the relationship between the requirements of industry and the teaching of mathematics.

R. G. MASSEY

BISRA Annual Report.

156 *pp.*

This Annual Report of the British Iron and Steel Research Association is naturally only of limited interest to operational research workers outside the steel industry, but there are 11 pages (out of about 80 which are not devoted to lists of committees, etc.) describing the work of the Operational Research Department. Much of this work was briefly described in the June, 1960, issue of the *Operational Research Quarterly* "Notes on Open Days at BISRA" and will not be referred to here. It is impossible for a report of this size to give more than a very brief outline of the year's work, and this is brought home by reading that the staff of BISRA is over 500 and its annual income nearly £1,000,000.

There are at least two topics, other than the work of the Operational Research Department, of interest outside the purely technical field—automation, and information handling. Several divisions of BISRA are working on automation problems: the Mechanical Working Division in connection with forging, the

Physics Department and Plant Engineering and Energy Division each in several fields. The latter division also studies information handling techniques, and the Tallimarker is referred to as a device which "consists essentially of a central information store which accumulates information about an ingot (or other material being processed) as it passes through the works, and finally prints out a complete record".

For operational research readers particularly, it would have been interesting, but more difficult than merely describing the work done, if the report had also described the approach to decisions about research effort being devoted to a particular project. Even with as large a research organization as BISRA, there must be many projects competing for attention and BISRA is as likely as any to make these decisions on some logical basis.

REX ACTON

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Abstracts

Bottlenecks in Shoe Production.

J. R. MANNING.

Journal of The British Boot and Shoe Institution, **9**, 5, *pp.* 200–208 and *p.* 214 (August, 1960).

A bottleneck is any factor which limits production. Bottlenecks can be overcome by (a) distension: a temporary expedient such as overtime working; or (b) destruction: a permanent increase in capacity such as a new factory; or (c) circumvention: reallocating resources to increase capacity. Bottlenecks are expressed as inequalities and their interactions can be explored by the techniques of linear programming. The equals sign represents a bottleneck condition, full use of some resource, and usually a tendency towards instability. The inequality represents spare capacity and room for manœuvre. Examples in the paper are all drawn from the shoe industry.

J. R. M.

Controlmanship.

L. A. STOCKDALE.

Automation and Automatic Equipment News, **6**, 2, *pp.* 72–74 (October, 1960).

Controlmanship is a short non-mathematical introduction to Cybernetics as applied to the study of large non-determinate systems. Reference is made to the black-box concept and to the basic assumption that some pattern or statistical determinacy must exist in the system under study. There is a suggestion as to the correspondence between subjective judgement, cybernetics and operational research. (Popular.)

W. M. P. H.

Experiences with the introduction of Office Automation.

E. WEINBERG (photographs by B. LAWRENCE).

Automation and Automatic Equipment News, **6**, 1, *pp.* 17–21 and *p.* 23 (September, 1960).

A review of the impact of electronic data processing on the employees of twenty private industry offices in the United States. It includes statistics of the regrouping of employees in affected units and the grouping of those employed in E.D.P., in both tabular and pictograph form. (The pictographs may be of particular interest to operational research workers.)

W. M. P. H.

Some Results for the Queue with Poisson Arrivals.

N. U. PRABHU.

Journal of the Royal Statistical Society, Series B, **22**, 1, *pp.* 104–107 (1960).

This paper is concerned with the queueing system in which (1) there is only one counter, (2) the customers arrive at random and are served in the order of arrival, and (3) the service-time distribution has a general form. The distribution of the time taken for a waiting-time x to reduce to zero for the first time is obtained, and from this are deducted the explicit expressions for the distributions of (i) the length of the busy period, (ii) the number of customers served during a busy period and (iii) the time taken for a queue of given length to disappear.

G. B. W.

The Busy Period in relation to the Single-Server Queueing Sytem with General Independent Arrivals and Erlangian Service-Time.

B. W. CONOLLY.

Journal of the Royal Statistical Society, Series B, **22**, 1, *pp.* 89–96 (1960).

A previous paper by this author concerned a single-server queueing system in which the arrivals took place in batches of fixed size k. The service time of an individual was assumed to be negative exponential, and the distribution of the intervals between the arrival of successive batches was arbitrary. This present paper deals with the distribution of the busy period of this system.

G. B. W.

A Queueing Problem in which the Arrival Times of Customers are Scheduled.

A. MERCER.

Journal of the Royal Statistical Society, Series B, **22**, 1, *pp.* 108–113 (1960).

The queueing problem considered is that in which (i) the customers are scheduled to arrive at equal time intervals but a customer may arrive at any time after the start of the interval during which he was scheduled to arrive, or may even not arrive at all; the lateness distribution is perfectly general: (ii) the total service-time of a customer is supposed to consist of a finite number of stages, such that the time spent in each of the stages are independent and identically distributed, and the probability of leaving a stage at any time depends on the time since the current scheduling interval began.

A method is discussed which enables the non-equilibrium distribution of the queue length at any time to be derived, and results for the equilibrium distribution are given. Most of the results relate to the case in which there is a single server but extensions of the argument for more than one server are outlined.

G. B. W.

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On the theory of Classical Regression and Double Sample Estimation.

B. D. TIKKIWAL.

Journal of the Royal Statistical Society, Series B, **22**, 1, *pp.* 131–138 (1960).

In some survey problems it is desired to estimate the mean value of some character of a population, and information on an auxiliary variable is available. Various classical results are examined and extended to cover a finite population.

G. B. W.

Regression Analysis when there is prior information about Supplementary Variables.

D. R. COX.

Journal of the Royal Statistical Society, Series B, **22**, 1, *pp.* 172–176 (1960).

For the problem of estimating the regression between two variables, precision can sometimes be increased by taking observations on a third variable. The paper contains some general discussion of this, with a note on the implications for experimental design. One simple special case is dealt with in which regression is required between two variables which are connected only through another random variable.

G. B. W.

Sociology and Management Studies.

J. H. SMITH.

British Journal of Sociology, XI (2) *pp.* 103–111 (*June*, 1960).

The paper claims that sociology should find its place as the main humanist discipline (or liberal element) in management studies. Reasons are given for the reluctance of British universities to include sociology in management courses. Exceptions to this rule are noted at Liverpool, Cardiff and L.S.E. This question is important now that consideration is being given to the establishment of business schools on the Harvard model. A course entitled "Industry and Society" is suggested with the following sub-divisions; (i) The Social Structure of Industry, (ii) Social Relations in the Enterprise, (iii) Industrialization and Social Change.

D. J. B.

Management Control.

V. W. OUBRIDGE.

The Manager, pp. 513–516, *p.* 551 (*July*, 1960).

An approach to the analysis of management organization and control which makes use of concepts taken from Cybernetics. Three forms of management are distinguished (administrative, executive and supervisory) and the relationship between these is discussed.

R. A. K. L.

A College Curriculum for Operations Research.

MARY E. MURPHY.

The Manager, pp. 288–290 (*April*, 1960).

The place and content of operational research courses in Management Studies at universities and institutes in the United States is discussed. The plea is made for greater uniformity in training for operational research in universities. The author outlines what she considers to be the essential requirements of a student preparing to enter the field of operational research.

R. A. K. L.

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